$\boldsymbol{\delta(\delta g)*}$ - Sets and Functions in Topological Spaces

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ABSTRACT

This study introduced the notion of $\delta(\delta g)*$ - set and functions in topological spaces (briefly TS). This proves that in TS, the δ-closure of a set is smaller than its $\delta(\delta g)*$ - closure while the δ-interior is generally larger than its $\delta(\delta g)*$ - interior. In addition, in the same space the $\delta(\delta g)*-$continuous functions, absolute-$\delta(\delta g)*$ - continuous functions and rs - $\delta(\delta g)*$ - continuous functions are introduced and investigated. Characterization and properties of these functions are also determined.

Keywords: *$\delta(\delta g)*$ - set, $\delta(\delta g)*$ - closure, $\delta(\delta g)*-$continuous functions, absolute-$\delta(\delta g)*$ - continuous functions, rs - $\delta(\delta g)*$ - continuous functions*

1. INTRODUCTION

Over the years, several types of closed and open sets have been introduced in an arbitrary topological space. In 1968, Velicko [24] introduced sets that are stronger than open sets called δ-open sets. On the other hand in 1970, Norman Levine [11] introduced and investigated the concepts of generalized closed (briefly, g-closed) sets. By combining the notions of δ-closedness and g-closedness, Julian Dontchev [3] proposed a class of generalized closed sets called δg-closed set in 1996. Later, Thivagar et.al [25] introduced a class that lies between the class of δ-closed sets and δg-closed sets called $\delta\hat{g}$-closed sets in 2010.

In 2012, the two authors Sudha R. and Sivakamasundari, K. [19] introduced and investigated another generalized closed set namely $\delta g*$ - closed set. Also, in 2014 K. Meena and K. Sivakamasundari [14] introduced a new class of generalized closed sets called $\delta(\delta g)*$ - closed set in topological spaces.

Along these concepts the author is highly motivated to introduce and investigate some properties of the notion of $\delta(\delta g)*-$closed set in topological spaces. This paper presents several characterization, properties and examples related to the new concepts.

2. $\boldsymbol{\delta(\delta g)*}$-Sets and Functions In Topological Spaces

This section introduces the concepts of δ(δg)*- closure, $\delta(\delta g)*$-interior of a set, $\delta(\delta g)*-$continuous, absolute-$\delta(\delta g)*-$continuous, and regular strongly- $\delta(\delta g)*-$continuous functions in TS. All throughout, (X, τ), (Y, τ), (Z, τ) are TS. Some basic properties, relationships and characterizations involving these sets are considered.

2.1 $\boldsymbol{\delta(\delta g)*}$ - Closure and $\boldsymbol{\delta(\delta g)*}$ - Interior of a Set

Definition 2.1 Let (X, τ) be a TS. Then,

(i) $\delta(\delta g)*-$*closure* of A denoted by $\delta(\delta g)*$ - *cl(A)* is the intersection of all δ(δg)*-closed sets in X containing A.

(ii) $\delta(\delta g)*-$*interior* of A denoted by $\delta(\delta g)*$ - int(A) is the union of all $\delta(\delta g)*$-open sets in X contained in A.

Remark 2.2 *Let (X, τ) be a topological space. For any $A \subseteq X$, $A \subseteq \delta(\delta g)^*$- cl(A) and $\delta(\delta g)^*$- int (A) $\subseteq$ A.*

Example 2.3 Let X = {a, b, c} and τ = {ϕ, X, {a, c}, {b}}. Then, the $\delta(\delta g)*-$closed sets in X are ϕ, X, {b} and {a, c} and the $\delta(\delta g)*-$open sets are ϕ, X, {a, c} and {b}. Thus, the $\delta(\delta g)*$ - cl({a}) = {a, c} and $\delta(\delta g)*$ - int({a}) = ϕ.

Remark 2.4 The *union of two $\delta(\delta g)^*$-open sets in X is not generally a $\delta(\delta g)^*$- open set in X.*

This section also presents some results about $\delta(\delta g)*$ - closure and $\delta(\delta g)*$ - interior of A. First, consider the following result.

Theorem 2.5 *Let A and B be nonempty subsets of X. If $A \neq \phi$, then $a \epsilon \delta(\delta g) * - cl(A)$ if and only if for every $\delta(\delta g) * -$ open set U with a ϵ U, U $\bigcap$ A $\neq \phi$.*

Proof: Let a ϵ $\delta(\delta g) * - cl(A)$ and let U be $\delta(\delta g) * -$ open set with a ϵ U. Suppose that U $\bigcap$ A $= \phi$. Then A $\subseteq$ U^c where U^c is $\delta(\delta g) * -$closed. Since a $\notin U^c$, a $\notin \delta(\delta g) * -cl\ (A)$. Thus we have a contradiction. Hence, U $\cap A \neq \phi$.

Conversely, assume that for every $\delta(\delta g) * -open\ set\ U$ with $a\, \epsilon\, U, U \cap A \neq \phi$. Suppose that $a \notin \delta(\delta g) * - cl\ (A)$. Then there exists a $\delta(\delta g) * -closed\ set\ F$ with $A \subseteq F$ and $a \notin F$. Thus $F^c \cap A = \phi$ and a $\epsilon\, F^c$. Since F^c is $\delta(\delta g) *$ - open, a contradiction to the assumption is obtained. Therefore, $a\, \epsilon\, \delta(\delta g) * cl\ (A)$.

Theorem 2.6 *Let (X, τ) be a topological space and A, B and F, be subsets of X.*

(i) *If A is $\tau - \delta(\delta g) * -closed$, then A= $\delta(\delta g) * -cl(A) = \delta(\delta g) * -cl\left(\delta(\delta g) * -cl(A)\right)$.*

(ii) *If A$\subseteq$ B, then $\delta(\delta g) * -cl(A) \subseteq \delta(\delta g) * -cl\ (B)$.*

(iii) $\delta(\delta g) * -cl(A) \subseteq \delta(\delta g) * -cl\left(\delta(\delta g) * -cl(A)\right)$.

(iv) $\delta(\delta g) * -cl(A) \cup \delta(\delta g) * -cl\ (B) \subseteq \delta(\delta g) * -cl(A \cup B)$.

Proof:

(i) By Remark 2.2, $A \subseteq \delta(\delta g) * -cl(A)$. Since A is $\tau - \delta(\delta g) * -closed$, by Definition 2.1(i), $\delta(\delta g) * -cl(A) \subseteq A$. Hence, A= $\delta(\delta g) * -cl(A)$. Consequently, $\delta(\delta g) * -cl(A) = \delta(\delta g) * -cl\left(\delta(\delta g) * -cl(A)\right)$.

(ii) Let $x \in \delta(\delta g) * -cl(A)$ and let U be $\delta(\delta g) *$ - open set with $x \in U$. By Theorem 2.5, U $\cap A \neq \phi$. Since $A \subseteq B$, it follows that U $\cap B \neq \phi$. Therefore, $x \in \delta(\delta g) * cl - (B)$ implying that $\delta(\delta g) * -cl(A) \subseteq \delta(\delta g) * -cl\ (B)$.

(iii) Let $x \in \delta(\delta g) * -cl(A)$ and F be any $\delta(\delta g) * -$ closed set such that $\delta(\delta g) * -cl(A) \subseteq F$. Thus $x \in F$. By Definition 2.1 (i), $x \in \delta(\delta g) * -cl\big(\delta(\delta g) * cl(A)\big)$.

(iv) Since A and B are contained in $A \cup$ B by (ii), it follows that $\delta(\delta g) * -cl(A) \subseteq \delta(\delta g) * -cl(A \cup B)$ and $\delta(\delta g) * -cl\ (B) \subseteq \delta(\delta g) * -cl(A \cup B)$. Therefore, $\delta(\delta g) * -cl(A) \cup \delta(\delta g) * -cl\ (B) \subseteq \delta(\delta g) * -cl(A \cup B)$.

Theorem 2.5 *Let (X, τ) be a topological space and A, B and F, be subsets of X.*

(i) *If A is* $\delta(\delta g)*-open$*, then* A= $\delta(\delta g)*-int(A) = \delta(\delta g)*-int\left(\delta(\delta g)*-int(A)\right)$.

(ii) $x \in \delta(\delta g)*-int(A)$ *if and only if there exists a* $\delta(\delta g)*-open$ *set U with* $x \in U \subseteq A$.

(iii) *If* $A \subseteq B$*, then* $\delta(\delta g)*-int(A) \subseteq \delta(\delta g)*-int\,(B)$.

Proof:

(i) Let A be a $\delta(\delta g)*-$open subset of x. Since $\delta(\delta g)*-int(A) \subseteq A$, it suffices to show that $A \subseteq \delta(\delta g)*-int(A)$. Suppose $x \notin \delta(\delta g)*-int(A)$. Then by Definition 2.1 (ii) , $x \notin O$ for any $\delta(\delta g)*-$open set $O \subseteq A$. Hence, in particular $x \in A$ since A is $\delta(\delta g)*-$open. Thus, $A \subseteq \delta(\delta g)*-$int(A). Therefore, A= $\delta(\delta g)*-$int(A). It follows that $\delta(\delta g)*-int(A) = \delta(\delta g)*-int\left(\delta(\delta g)*-int(A)\right)$.

(ii) Let $x \in \delta(\delta g)*-int(A)$. Then by Definition of interior, $x \in U$ for some U- $\delta(\delta g)*-open$ set U with $U \subseteq A$.
The converse follows the definition 2.1(ii).

(iii) Let $A \subseteq B$. Suppose $x \in \delta(\delta g)*-int(A)$. By (ii), there exists a $\delta(\delta g)*-open$ set U with $x \in U \subseteq A$. Since $A \subseteq B$, there exists a $\delta(\delta g)*-$open set U with $x \in U \subseteq B$. Thus, $x \in \delta(\delta g)*-int(B)$. Therefore, $\delta(\delta g)*-int(A) \subseteq \delta(\delta g)*-int\,(B)$.

Theorem 2.6 *Let* $A \subseteq X$*. Then* $\delta(\delta g)*-int(A) = x \setminus [\delta(\delta g)*-cl\,(x\setminus A)]$.

Proof: Suppose $x \in \delta(\delta g)*-int(A)$. Then there exists a $\delta(\delta g)*-open$ set U with $x \epsilon U \subseteq A$. Hence, there exist a $\delta(\delta g)*-closed$ set X\ U with $x \in X\setminus U \supseteq X\setminus A$. This implies that $x \notin \delta(\delta g)*-cl(X\setminus A)$. Hence, $x \in X\setminus\delta(\delta g)*-cl(X\setminus A)$. Thus, $\delta(\delta g)*-int(A) \subseteq X\setminus[\,\delta(\delta g)*-cl\,(X\setminus A)]$.

Conversely, let $x \in X\setminus\delta(\delta g)*-cl(X\setminus A)$. Then $x \notin \delta(\delta g)*-cl(X\setminus A)$. This implies that there exists a $\delta(\delta g)*-$closed set F containing X \ A such that $x \notin F$. Hence, there exists a $\delta(\delta g)*-open$ set X \ F with $x \in X\setminus F \subseteq A$. It follows that $x \in \delta(\delta g)*-int(A)$. Therefore, $\delta(\delta g)*-int(A) \supseteq X \setminus [\delta(\delta g)*-cl(X\setminus A)]$ and so, equality follows.

The next Corollary follows immediately from Theorem 2.6.

Corollary 2.7 *Let* $A \subseteq X$*. Then* $\delta(\delta g)*-cl(A) = X \setminus [\delta(\delta g)*-int\,(X\setminus A)]$.

3. $\boldsymbol{\delta(\delta g)*-Continuous\ Functions}$

This section gives some properties of $\delta(\delta g)*-$continous functions.

Definition 3.1 Let (X, τ_X) and (Y, τ_Y) be spaces. A function f: X→ Y is said to be $\delta(\delta g)*-continous$, if the inverse image of each open set in Y is $\delta(\delta g)*-open$ in X.

Example 3.2 Let X= {a, b, c} and Y={u, v}. Consider the topologies $\tau_X=\{\phi, X, \{b\}, \{a,c\}\}$, and $\tau_Y=\{\phi, Y, \{u\}\}$. Then $\delta(\delta g)*-closed$ sets are $\phi, X, \{b\}$, and $\{a,c\}$ and $\delta(\delta g)*-open$ sets in X are $\phi, X, \{b\}, and\ \{a,c\}$.

Let f: (X, τ_X) → (Y, τ_Y) be defined by $f(a)= f(c) = \{u\}$ and $f(b) = \{u\}$. Then f is τ-$\delta(\delta g)*$-continuous in X since $f^{-1}(\phi)= \phi$, $f^{-1}(Y)= X$ and $f^{-1}(\{u\})= \{a,c\}$ where ϕ, X and {a,c} are $\delta(\delta g)*-open$ sets in X.

Therefore, f: (X, τ_X) → (Y, τ_Y) is a $\delta(\delta g)*-continous\ function$.

Theorem 3.3 *If f: X→ Y is $\delta(\delta g)*-continuous$ and g: Y→ Z is continuous, then g o f : X→ Z is $\delta(\delta g)*-continuous$.*

Proof: Let U be open in Z. Then by continuity of g, $g^{-1}(U)$ is open in Y. Since f is $\delta(\delta g)*-continous$, $f^{-1}(g^{-1}(U))=(g\ o\ f)^{-1}(U)$ is $\delta(\delta g)*-open$ in X. Therefore, $g\ o\ f$ is $\delta(\delta g)*-continous$.

Remark 3.4 *The composition of two $\delta(\delta g)*-continous\ functions$ need not be $\delta(\delta g)*-continous$.*

To see this, let X={a, b, c}, Y=Z={u, v, w} with their respective topologies τ_X={ $\phi, X, \{a\}, \{b\}, \{a,b\}$}, and τ_Y={ $\phi, Y, \{u\}$}, and τ_Z={ $\phi, Z, \{u,w\}$}.

Theorem 3.5 *A function f: X→ Y is $\delta(\delta g)*-continous$ if and only if the inverse image of each closed set in Y is $\delta(\delta g)*-closed$ in X.*

Proof: Let F be any closed set in Y. Then F^c is open in Y. Since f is $\delta(\delta g)*-continous$, $f^{-1}(F^c)$ is $\delta(\delta g)*-$open in X. Now, by the existing theorem $f^{-1}(Y\backslash F) = f^{-1}(Y)\backslash f^{-1}(F)$. Hence, $f^{-1}(F)$ is $\delta(\delta g)*-$closed in X.

Let U open in Y. Then U^c is closed in Y. By Assumption, $f^{-1}(U^c)$ is $\delta(\delta g)*-$closed in X. By this theorem $f^{-1}(Y\backslash U) = f^{-1}(Y)\backslash f^{-1}(U)$. Hence, $f^{-1}(U)$ is $\delta(\delta g)*-$open. Therefore, f is $\delta(\delta g)*-continous$.

Theorem 3.6 *If f: X → Y is $\delta(\delta g)*-continous$, then f($\delta(\delta g)*-cl(A) \subseteq cl(f(A))$ for every $A \subseteq X$.*

Proof: Let $A \subseteq$ X and let x $\epsilon\ \delta(\delta g)*-cl(A)$. Suppose that O is an open set in Y with $f(x)\ \epsilon\ O$. Since f is $\delta(\delta g)*-$continuous, $f^{-1}(O)$ is $\delta(\delta g)*-$open in X where $x\ \epsilon\ , f^{-1}(O)$. Hence, by Theorem 2.5, $f^{-1}(O)\ \cap A \neq \phi$. It follows that $\phi \neq f\left(, f^{-1}(O)\right) \cap A \subseteq f\left(f^{-1}(O)\right) \cap f(A) \subseteq O \cap f(A)$. Thus, $O\ \cap f(A) \neq \phi$. Therefore, by Theorem 2.5, $f(x)\ \epsilon\ cl - f(A)$.

Theorem 3.7 *If f: X→ Y is $\delta(\delta g)*-continous$, then $(\delta(\delta g)*-cl(f^{-1}(B) \subseteq f^{-1}(cl(B))$ for every $B \subseteq Y$.*

Proof: Let f: X→ Y is $\delta(\delta g)*-continous$, and $B \subseteq$ Y. By Theorem 3.6, f($\delta(\delta g)*-cl\left(\ f^{-1}(B)\right) \subseteq cl\ (f\left(f^{-1}(B)\right) \subseteq cl(B)$. Therefore, $\delta(\delta g)*-(cl\left(\ f^{-1}(B)\right) \subseteq \left(f^{-1}cl(B)\right)$.

Remark 3.8: *The converse of Theorem 3.6 and 3.7 are not true.*

Theorem 3.9 *If f: X→ Y is $\delta(\delta g)*-continous\ function$, then for every $x \epsilon$ X and every open set V in Y containing f(x), then there exists a $\delta(\delta g)*-open\ set$ O in X such that $x \epsilon$ O and f(O) $\subseteq V$.*

Proof: Let x ∈ X and V be an open set in Y containing f(x). Since f is $\delta(\delta g)*-continuous$, f^{-1}(V) is $\delta(\delta g)*-open$ in X. Let O = f^{-1} (V). Thus, $x \in O$ and by theorem $f(O) = f(f^{-1}(V)) \subseteq V$.

Remark 3.8: *The converse of Theorem 3.9 is not true.*

4. Regular Strongly- $\boldsymbol{\delta(\delta g)*-Continous}$ Functions

Definition 4.1 Let (X, τ_X) and (Y, τ_Y) be spaces. A function f: X→ Y is said to be $regular\ strongly-\delta(\delta g)*-$continous, if the inverse image of each $\delta(\delta g)*-open$ set in Y is open in X.

Example 4.2 Let X= {a, b, c} and Y={u, v, w}. Consider the topologies τ_X=$\{\phi, X, \{a\}, \{b, c\}\}$, and τ_Y= $\{\ \phi, Y, \{u, v\}, \{u, w\}\}$. Then, $\delta(\delta g)*-open$ sets in Y are $\phi, Y, \{v\}, and\ \{u, w\}$.

Let f: (X, τ_X) → (Y, τ_Y) be defined by $f(a) = v,\ f(b) = u$ and $f(c) = w$ Then f is rs- $\delta(\delta g)*$-continuous in X since f^{-1} (ϕ)= ϕ, f^{-1} (Y)= X and f^{-1} $(\{u, w\})$= $\{b, c\}$ and $f-1$ $(\{v\}) = \{a\}$ where ϕ, X,{b,c} and {a} are $open$ sets in X.

Therefore, f: (X, τ_X) → (Y, τ_Y) is a $rs-\delta(\delta g)*-continous\ function.$

Theorem 4.3 *A function f: X→ Y is an rs-$\delta(\delta g)*-continous$ if and only if f^{-1} (F) is closed for every $\delta(\delta g)*-$closed set F in Y.*

Proof: Let F be $\delta(\delta g)*-closed$ set in Y. Then Y\F is $\delta(\delta g)*-open$ in Y. Since f is $rs-\delta(\delta g)*-continuous$, $f^{-1}(Y\backslash F)$ is open in X. However, by *f^{-1}(Y\F) = f^{-1}(Y)\f-1(F)*. Hence, f^{-1}(Y) is closed.

Let O be $\delta(\delta g)*-open$ set in Y. Then Y\O is $\delta(\delta g)*-closed$ in Y. By assumption, *f^{-1}(Y\O)* is closed. By *f^{-1}(Y\O) = f^{-1}(Y)\f-1(O)* is closed. Therefore, *f^{-1}(O)* is open in X implying that *f* is *rs*- $\delta(\delta g)*-continuous$.

Theorem 4.3 *If f: X→ Y is an rs-$\delta(\delta g)*-continous$ function, then for every $x \in X$ and for every $\delta(\delta g)*-$open set V in Y containing f(x), there exists an open set O in X such that $x \in O$ and f(O)$\subseteq V$.*

Proof: Let *$x \epsilon X$* and *V* be a $\delta(\delta g)*-open$ set containing *f(x)*. Since *f* is *rs*-$\delta(\delta g)*-continous$, *$f^{-1}$(V)* is open. Let *O = f^{-1}(V)*. Thus, *$x \epsilon O$* and by f(*O)* =*f(f^{-1}(V))* ⊆ *(V).*

5. Absolute- $\delta(\delta g)*-Continous$ Functions

Definition 5.1 Let (X, τ_X) and (Y, τ_Y) be spaces. A function *f: X*→ Y is said to be $absolute-\delta(\delta g)*-$*continous*, if the inverse image of each open set in Y is $\delta(\delta g)*-open$ in Y is open in X.

Example 5.2 Let X= {a, b, c} and Y={u, v, w}. Consider the topologies τ_X={$\phi, X, \{a\}, \{b, c\}$}, and τ_Y= { $\phi, Y, \{u, v\}, \{u, w\}$}. Then, $\delta(\delta g)*-open$ set in Y are $\phi, Y, \{v\}, and\ \{u, w\}$. Also, τ_X={$\phi, X, \{a, b\}, \{a, c\}$} and τ_Y= { $\phi, Y, \{u\}$}. Then, the $\delta(\delta g)*-open$ set in X are $\phi, X, \{b\}, \{a, c\}$.

Let *f: (X, τ_X)* → (Y, τ_Y) be defined by f*(a)=u*, *f(b)* = v and *f(c)*= w. Then *f* is $absolute$-$\delta(\delta g)*$-*continuous* in X since f^{-1} (ϕ)= ϕ, f^{-1} (Y)= X and f^{-1} ($\{u, w\}$)= $\{a, c\}$ and f^{-1} ($\{v\}$)= $\{b\}$ where ϕ, X, {a,c} and {b} are $\delta(\delta g)*-open$ in X.

Theorem 5.3 *A function f: X*→ Y *is an absolute-*$\delta(\delta g)*-$*continuous function if and only if* f^{-1}*(B) is* $\delta(\delta g)*-$*closed for every* $\delta(\delta g)*-$*closed set B in Y.*

Proof: Let *f* be *absolute-* $\delta(\delta g)*-continuous$ function and be a $\delta(\delta g)*-closed$ set in Y. Then Y\B is $\delta(\delta g)*-open$ in Y. By Definition 5.1, by f^{-1}*(Y\B)* is $\delta(\delta g)*-open$ in X. By f^{-1}*(Y\B)* = f^{-1}*(Y)* \ f^{-1} *(B)* is $\delta(\delta g)*-open$ in X. Hence, f^{-1}*(B)* is $\delta(\delta g)*-$ closed in X.

Let O be $\delta(\delta g)*-open$ set in Y. Then $Y \setminus O$ is $\delta(\delta g)*-closed$ in Y. By, *[X \ f^{-1}(O)]* = f^{-1}*(Y)* \ *f1(O).* Thus, *[X \ f^{-1}(O)]* is $\delta(\delta g)*-$closed in X implying that f^{-1}*(O)* is $\delta(\delta g)*-open$ in X. Therefore, *f* is *absolute-* $\delta(\delta g)*-continuous$.

Theorem 5.4 *If f: X*→ Y *be an absolute-*$\delta(\delta g)*-$*continous function. The following hold:*

(i) $f(\delta(\delta g)*)-cl(A) \subseteq \delta(\delta g)*-cl\left(f(A)\right)\ for\ every\ A \subseteq X.$

(ii) $\delta(\delta g)*-cl\left(f^{-1}(B)\right) \subseteq f^{-1}(\delta(\delta g)*)-cl\ (B)\ for\ every\ B \subseteq Y.$

Proof: (a) Let $A \subseteq X$ and let *f(x)* ∈ $f(\delta(\delta g)*)-cl(A)$. Suppose that O is a $\delta(\delta g)*-open$ set in Y with $f(x) \in O$. Since f is *absolute-* $\delta(\delta g)*-continuous$, *f(O)* is $\delta(\delta g)*-$ open in Y with $x \in f^{-1}(O)$. By Theorem, $f^{-1}(O) \cap A \neq \phi$ since $x \epsilon \delta(\delta g)*-cl(A)$. It follows that, $\phi \neq$ f($f^{-1}(O)$) ∩ A *f(*$f^{-1}(O)$) ∩ $f(A) \subseteq O \cap f(A)$. Thus, $O \cap f(A) \neq \phi$. Therefore, by Theorem *f(x)* $\epsilon\ \delta(\delta g)*-\left(cl\left(f(A)\right)\right)$.

(b) Let $B \subseteq Y$ and $A = f^{-1}(B)$. By (a),

$$f(\delta(\delta g)*)-cl(A) = f(\ \delta(\delta g)*)-cl(f^{-1}(B))$$
$$\subseteq \delta(\delta g)*-cl\left(f(f^{-1}(B)\right)$$
$$\subseteq \delta(\delta g)*-cl(B).$$

Hence, $(\delta(\delta g)*)-cl(A) \subseteq f^{-1}(f(\ \delta(\delta g)*)-cl(B) \subseteq f^{-1}(\delta(\delta g)*)-cl(B)$. Therefore, $(\delta(\delta g)*)-cl\left(f^{-1}(B)\right) \subseteq f^{-1}(f(\ \delta(\delta g)*)-cl(B)$.

Theorem 5.5 *If f: X*→ Y *is an absolute-*$\delta(\delta g)*-$*continuous function, then for every x* ∈ *X and for every* $\delta(\delta g)*-$*open set V in Y containing f(x), there exists a* $\delta(\delta g)*-$*open set O in X such that x* ∈ O *and f(O)* ⊆ V.

Proof: Let $x \in X$ and V be a $\delta(\delta g)*-open$ set containing *f(x)*. Since f is *absolute*- $\delta(\delta g)*-continuous$, $f^{-1}(V)$ is $\delta(\delta g)*-open$. Let $O = f^{-1}(V)$. Thus, $x \in O$ and by Theorem, $f(O) = f(f^{-1}(V)) \subseteq V$.

Theorem 5.6 *If f: X→ Y and g: Y → Z are absolute-*$\delta(\delta g)*-$*continuous functions, then g o f: X→ Z is absolute* $\delta(\delta g)*-$*continuous.*
Proof: Let O be a $\delta(\delta g)*-open$ set in Z. Since g is *absolute*- $\delta(\delta g)*-continuous$, $g^{-1}(O)$ is $\delta(\delta g)*-open$ set in Y. Hence, $f^{-1}\left(g^{-1}(O)\right)$ is $\delta(\delta g)*-open$ in X since f is *absolute*-$\delta(\delta g)-\ continuous$. Thus, $(g\ o\ f)^{-1}(O)= f^{-1}\left(g^{-1}(O)\right)$ is $\delta(\delta g)*-open$ in X. Therefore, by Theorem 5.6, $g\ o\ f$ is *absolute*-$\delta(\delta g)*-continuous$.

REFERENCES

[1] Baculta, J.J., *Regular Generalized Star* $\beta-sets$ *in Generalized, Bigeneralized and Generalized Fuzzy Topological Spaces* Ph. D. Thesis. MSU-Iligan Institute of Technology, Iligan City. (2015)

[2] Dontchev, J. I. Arokiarani, I. and Balanchandran, K., *On Generalised* δ*-Closed Sets and Almost Weakly Hausdorff Spaces.*, Topology Atlas, (1997).

[3] Dontchev, J. and Ganster. M., *On* $\delta-Generalised$ *Closed Sets and T ¾ Spaces*, Mem. Fac. Sci. Kochi. Univ. Math. 17 (1996), 5-31.

[4] Dontchev, J. and Noiri, T., *Quasi-normal Spaces and* $\pi g-Closed\ Sets$. Acta Math. Hungar 89 (3)(2000):211-219.

[5]Dugunji, J., *Topology*. New Delhi Prentice Hall of India Private Ltd., (1975).

[6] Elvina, M.L., *(gs)*-Closed Sets in Topological Spaces.* International Journal of Mathematics Trends and Technology, (7)(2014), 83-93.

[7]Gnanambal, Y., *On Generalized Pre-regular Sets in Topological Spaces.*, Ind. J.Pure.Appl.Math 28(3)(1997):351-360.

[8] Jafari, S., Noiri, T., Rajesh, N. and Thivagar, M.L., *Another Generalisation of Closed Sets.*, Kochi.J. Math(3)(2008), 25-38.

[9] Janaki, C., *Studies on g-closed Sets in Topological.*, Ph. D. Thesis Bharathiar University Coimbatore, India (1999).

[10] Levine, N., *Semi-open Sets and Semi-continuity in Topological spaces.*, Amer. Math. Monthly., 70(1963), 36-41.

[11] Levine, N. *Generalized Closed sets in Topology.*, Rend. Circ. Math. Palermo, 19(1970), 89-96.

[12] Maki, H., Devi, R. and Balanchandran, K., *Associated Topologies of generalized-closed Sets and* $\alpha - closed\ sets\ and\ \alpha - generalized\ closed\ sets$*.,* Mem. Fac. Sci. Kochi Univ. (Math)15(1994), 51-63.

[13] Maki, H., Umehara, J. and Noiri, T., *Every Topological Space is Pre-T1/2.*, Mem.Fac.Sci. Kochi Univ Ser. Alath 17(1996), 33-42.

[14] Meena, K. and Sivakamasundari, K., $\delta(\delta g) * -Closed\ Set\ in\ Topological\ Spaces$. Vol.3(7)(2014), 14749-14753.

[15] Palaniappan, N. and Rao K.C., *Regular Generalized Closed Sets.*, Kyungpook Math. J 33(1993), 211-219.

[16] Pushpalatha, A. and Anitha, K., *Definition Bank in General Topology and the Properties g*s-Closed Sets in Topological Spaces,* Int.J. Contemp.Math Sciences, (6)(2011), 917-929.

[17] Sarsak, M.S. and Rajesh, N., *Generalized Semi-pre-Closed Sets.* International Mathematical Forum., 5(12)(2010):573-578.

[18] Shylac I.M.T., and Thangavelu, P., *On Regular Pre-semi Closed Sets in Topological Spaces.,*KBM journal of Mathematical Sciences and Computer Applications, (2010)1(1), 9-17.

[19] Sudha, R. and Sivakamasundari, K., $\delta g * -Closed\ Sets\ in\ Topological\ Spaces$*.,* International Journal of Mathematical Archive, (2012)3(3), 1222-1230.

[20] Thivagar, M.L., Meeradevi, B. and Hatir, E., $\delta\hat{g}$*-Closed Sets in Topological Spaces.*, Gen Math.Notes, (2)(2010), 17-25.

[21] Thivagar, M.L., Meeradevi, B. and Hatir, E., $\delta - Closed\ Sets\ and\ \delta g - Closed\ Sets\ called\ \delta\hat{g}$*-Closed Sets* ., Gen Math.Notes, Vol.1, No.2, (2010), 17-25.

[22] Vadivel, A. and Vairamanickam, K., $rg\alpha - Closed\ Sets\ and\ rg\alpha - Open\ Sets\ in\ Topological\ Spaces$., Int.J.Math Analysis (2010) 3(37);1803-1819.

[23] Veerakumar, M.K.R.S., $\hat{g} - Closed\ Sets\ in\ Topological\ Spaces$*.,* Bull. Allah.Math.soc., (18)(2003), 99-112.

[24] Velicko, N.V., $H - Closed\ Topological\ Spaces$*,* Amer., Math Soc. Transl., 78(1968), 103-118.