The Some Properties of Beta Hat Generalized Closed Set in Generalized Topological Spaces

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ABSTRACT

This study introduced the concept of beta hat generalized closed set in generalized topological spaces. It also investigated related concepts such as μ-$\hat{\beta}g$-interior and μ-$\hat{\beta}g$-closure of a set. This paper proved that every μ-open (resp. μ-closed) set is μ-$\hat{\beta}g$-open (resp. μ-$\hat{\beta}g$-closed). It is also shown that the family of all μ-$\hat{\beta}g$-open subsets in X does not always form a generalized topology in X. Moreover, μ-$\hat{\beta}g$-closure of set is smaller than μ-closure while the μ-$\hat{\beta}g$-interior is generally larger than its μ-interior.

Keywords: ***closed set, generalized topological spaces, beta hat generalized closed set***

1. INTRODUCTION

From the time when Pure Mathematics has given significance, many diverged concepts on sets has been introduced by our Mathematicians across the globe. One of the most important set is the closed set which play a very essential role in topology. As years passed, diverse forms of closed sets have been studied, characterized and introduced in an arbitrary topological space. One of which is the generalized closed (briefly g-closed) set which was pioneered by Levine [1] in the year 1970. In his study, he compared closure of a subset with its open supersets and discussed the properties of sets, closed and open maps, compactness, and normal and separation axioms. This paved way to more developments in general topology. In recent years, many topologists studied extensively on the use of closed sets. In 2012, K. Kannan and N. Nagaveni [2] further investigated a new class of closed sets called beta hat generalized closed (briefly $\hat{\beta}g$-closed) set and introduced its properties on topological spaces. In 2001, Császár [3] introduced the concept of generalized topological spaces (briefly GTS). The results gathered were utilized to further developments in GTS. Throughout this paper, we extended the $\hat{\beta}g$-closed sets to GTS wherein various properties and characterization associated to these were widely-investigated.

This paper introduced and investigated the concept of beta hat generalized closed set in generalized topological space. Specifically, the $\hat{\beta}g$-closed sets in GTS were defined and characterized. Additionally, some properties of $\hat{\beta}g$-closure and $\hat{\beta}g$-interior of a set in GTS were defined and investigated.

2. On μ-$\hat{\beta}g$-CLOSED SETS IN GTS

In 2012, Kannan and Navageni introduced $\hat{\beta}g$-closed sets in topological space [2]. In this section, this concept is defined in GTS. This section introduces the concepts of μ-$\hat{\beta}g$-closed sets,

μ-$\hat{\beta}g$-open sets, μ-$\hat{\beta}g$-closure and μ-$\hat{\beta}g$-interior of a set in GTS. Some basic properties, relationships and characterizations involving these sets are considered.

All throughout, X denotes the GTS (X,μ) unless otherwise specified.

Definition 2.1 Let (X, μ) be a GTS and $A \subseteq X$. Then A is called *μ-beta hat generalized closed* (briefly μ-$\hat{\beta}g$-closed) *set* if $c_\mu(i_\mu(c_\mu(A))) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in X. The complement of μ-$\hat{\beta}g$-closed set is *μ-beta hat generalized open* (briefly μ-$\hat{\beta}g$-open) *set*.

Example 2.2 Let $X = \{a, b, c\}$. Consider the generalized topology $\mu = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$. Then the μ-closed sets in X are X, $\emptyset$, $\{b,c\}$, $\{a,b\}$, and $\{b\}$. Thus the μ-$\hat{\beta}g$-closed sets in X are $\emptyset$, X, $\{b\}$, $\{a,b\}$ and $\{b,c\}$. Hence, the μ-$\hat{\beta}g$-open sets in X are X, $\emptyset$, $\{a,c\}$, $\{c\}$ and $\{a\}$.

The next theorem characterizes a μ-$\hat{\beta}g$-open set.

Theorem 2.3 *Let (X, μ) be a GTS and $A \subseteq X$. Then A is a μ-$\hat{\beta}g$-open set if and only if $U \subseteq i_\mu(c_\mu(i_\mu(A)))$ whenever $U \subseteq A$ and U is μ-closed in X.*

Proof: Suppose that A is μ-$\hat{\beta}g$-open in X. Let $U \subseteq A$ where U is μ-closed in X. Then $X \setminus A \subseteq X \setminus U$ where $X \setminus U$ is μ-open set in X. Since $X \setminus A$ is μ-$\hat{\beta}g$-closed in X, $c_\mu(i_\mu(c_\mu(X \setminus A))) \subseteq (X \setminus U)$. Thus, $(X \setminus U)^c \subseteq (c_\mu(i_\mu(c_\mu(X \setminus A))))^c$. It implies that $U \subseteq i_\mu(c_\mu(i_\mu(A)))$.
□

Conversely, let $X \setminus A \subseteq U$ where U is μ-open set in X. Then $X \setminus U$ is μ-closed in X with $X \setminus U \subseteq A$. By assumption, $X \setminus U \subseteq i_\mu(c_\mu(i_\mu(A)))$. Hence, $c_\mu(i_\mu(c_\mu(X \setminus A))) \subseteq U$. Therefore, $X \setminus A$ is μ-$\hat{\beta}g$-closed in X which implies that A is μ-$\hat{\beta}g$-open in X.

Theorem 2.4 *The union of two μ-$\hat{\beta}g$-closed subsets of X is also the μ-$\hat{\beta}g$-closed subset of X.*

Proof: Assume that A and B are μ-$\hat{\beta}g$-closed subsets in X. Let U be a μ-open set in X such that $A \cup B \subset U$. Then, $A \subseteq U$ and $B \subset U$. Since A and B are μ-$\hat{\beta}g$-closed, $c_\mu(i_\mu(c_\mu(A))) \subseteq U$ and $c_\mu(i_\mu(c_\mu(B))) \subseteq U$. Thus, $c_\mu(i_\mu(c_\mu(A))) \cup c_\mu(i_\mu(c_\mu(B))) \subseteq U$. Since, $c_\mu(i_\mu(c_\mu(A \cup B))) \subseteq c_\mu(i_\mu(c_\mu(A))) \cup c_\mu(i_\mu(c_\mu(B)))$ it follows that $c_\mu(i_\mu(c_\mu(A \cup B))) \subseteq U$. Therefore, $A \cup B$ is μ-$\hat{\beta}g$-closed.
□

Remark 2.5 *The intersection of two μ-$\hat{\beta}g$-closed subsets of X is generally not μ-$\hat{\beta}g$-closed subset of X.*

To see this, consider the next example.

Example 2.6 Let $X = \{a,b,c\}$ and consider the generalized topology $\mu = \{\emptyset, X, \{a\}\}$. Then, the μ-closed subsets are $X, \emptyset$, and $\{b,c\}$. Note that $\{a,b\}$ and $\{a,c\}$ are μ-$\hat{\beta}g$-closed sets but $\{a, b\} \cap \{a, c\} = \{a\}$ is not μ-$\hat{\beta}g$-closed set in X.

Theorem 2.7 *If a subset A of X is μ-$\hat{\beta}g$-closed set in X, then $c_\mu(i_\mu(c_\mu(A))) \setminus A$ contains no non-empty μ-closed set.*

Proof: Suppose that A is μ-$\hat{\beta}g$-closed set in X and F is a μ-closed set such that $F \subseteq c_\mu(i_\mu(c_\mu(A))) \setminus A$. Then $F \subseteq c_\mu(i_\mu(c_\mu(A)))$ and $F \subseteq X \setminus A$. Thus, $A \subseteq X \setminus F$ where $X \setminus F$ is μ-open set in X. Since A is μ-$\hat{\beta}g$ -closed, $c_\mu(i_\mu(c_\mu(A))) \subseteq X \setminus F$. Thus, $F \subseteq X \setminus c_\mu(i_\mu(c_\mu(A)))$. Hence, $F \subseteq c_\mu(i_\mu(c_\mu(A))) \cap X \setminus c_\mu(i_\mu(c_\mu(A))) = \varnothing$. Therefore, $F = \varnothing$. Consequently, $c_\mu(i_\mu(c_\mu(A))) \setminus A$ contains no non-empty μ-closed set. □

Remark 2.8 *The converse of Theorem 2.7 is not necessarily true as shown in the next example.*

Example 2.9 Let $X = \{a, b, c, d\}$. Consider the generalized topology $\mu = \{\varnothing, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then the μ-closed sets in X are X, $\{c, d\}$, $\{a, d\}$ and $\{d\}$. The μ-$\hat{\beta}g$-closed sets in X are X, $\{d\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, c, d\}$ and $\{b, c, d\}$. Consider set $\{b\}$ which is not μ-$\hat{\beta}g$-closed. Note that $c_\mu(i_\mu(c_\mu(\{d\}))) = X$. Observe that $c_\mu(i_\mu(c_\mu(\{d\}))) \setminus \{d\} = \{a, c, d\}$ which is not μ-closed.

Theorem 2.10 *Every μ-closed set is μ-$\hat{\beta}g$-closed.*

Proof: Let A be any μ-closed set in X such that $A \subseteq U$ where U is μ-open. Since A is μ-closed, $A = c_\mu(A)$. Now, $i_\mu(c_\mu(A)) \subseteq c_\mu(A)$. Thus, $c_\mu(i_\mu(c_\mu(A))) \subseteq c_\mu(c_\mu(A))$. By Theorem 1.7.1 (vi), $c_\mu(c_\mu(A)) = c_\mu(A)$. Hence, $c_\mu(i_\mu(c_\mu(A))) \subseteq c_\mu(A) = A \subseteq U$. This implies that $c_\mu(i_\mu(c_\mu(A))) \subseteq U$. Therefore, A is μ-$\hat{\beta}g$-closed.

□

Remark 2.11 *The converse of the Theorem 2.10 is not necessarily true as shown in the following example.*

Example 2.12 Let $X = \{a, b, c, d\}$. Consider the generalized topology $\mu = \{\varnothing, X, \{b,c\}, \{a,c,d\}\}$. Then the μ-closed sets in X are X, $\varnothing$, $\{a,d\}$ and $\{b\}$. The μ-$\hat{\beta}g$-closed sets in X are $\varnothing$, X, $\{a\}$, $\{b\}$, $\{d\}$ and $\{a,d\}$ and the μ-$\hat{\beta}g$-open sets in X are X, $\varnothing$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,c\}$ and $\{b,c\}$. Note that the set $\{a\}$ is μ-$\hat{\beta}g$-closed since $c_\mu(i_\mu(c_\mu(\{a\}))) = \varnothing$ is a subset of X and $\{a,c,d\}$ where X and $\{a,c,d\}$ are μ-open sets. However, $\{a\}$ and $\{d\}$ is not μ-closed set in X.

The next corollary follows from Theorem 2.10.

Corollary 2.13 *Every μ-regular closed set in X is μ-$\hat{\beta}g$-closed.*

Remark 2.14 *The converse of Corollary 2.13 need not be true as seen from the following example.*

Example 2.15 Consider $X = \{a, b, c, d\}$ and the generalized topology $\mu = \{\varnothing, X, \{b, c\}, \{a, c, d\}\}$. Then the μ-closed sets in X are X, $\varnothing$, $\{a, d\}$ and $\{b\}$. The μ-$\hat{\beta}g$-closed sets in X are $\varnothing$, X, $\{a\}$, $\{b\}$, $\{d\}$ and $\{a, d\}$. Also, the μ-regular closed in X are $\varnothing$ and X. Observe that $\{a\}$, $\{b\}$, $\{d\}$ and $\{a, d\}$ are μ-$\hat{\beta}g$-closed in X but not μ-regular closed in X.

The next corollary follows from Theorem 2.10.

Corollary 2.16 *Every μ-open set is μ-$\hat{\beta}g$-open.*

Remark 2.17 *The converse of Corollary 2.16 is not necessarily true as shown in the following example.*

Example 2.18 Consider Example 2.15 with μ-open sets $\varnothing$, X, $\{b,c\}$ and $\{a,c,d\}$. Then the μ-$\hat{\beta}g$-open sets in X are X, $\varnothing$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,c\}$ and $\{b,c\}$. Note that the sets $\{b,c,d\}$ and $\{a,b,c\}$ is μ-$\hat{\beta}g$-open. However, the sets $\{b,c,d\}$ and $\{a,b,c\}$ are not μ-open sets in X.

Theorem 2.19 *The empty set is μ- β g-open.*

Proof: Suppose $\varnothing$ is not μ- β g-open. Then there exists a μ-open set $U \subseteq \varnothing$ such that $U \nsubseteq i_\mu(c_\mu(i_\mu(A)))$. This is a contradiction. Therefore, the empty set is μ- β g-open.

□

The next corollary follows from Theorem 2.19.

Corollary 2.20 *X is μ- β g-closed.*

Theorem 2.21 *Let (X, μ) be a GTS. Then every singleton is either μ-closed or μ-$\hat{\beta}g$-open.*

Proof: Suppose $X \setminus \{x\}$ is not μ-open. If X is μ-open, then X is the only μ-open set containing $X \setminus \{x\}$. This implies $c_\mu(i_\mu(c_\mu(X \setminus \{x\}))) \subseteq X$. Hence $X \setminus \{x\}$ is a μ-$\hat{\beta}g$-closed set in X.

□

Theorem 2.22 *If A is μ-regular open and μ-rg-closed, then A is μ-$\hat{\beta}g$-closed set in X.*

Proof: Let A be μ-regular open and μ-rg-closed in X. We prove that A is a μ-$\hat{\beta}g$-closed set in X. Let U be any μ-open set in X such that $A \subseteq U$. Since A is μ-regular open and μ-rg-closed, we have $c_\mu(A) \subseteq A$. Now, $i_\mu(c_\mu(A)) \subseteq i_\mu(A)$. Since A is regular open, it follows that A is open. Thus $i_\mu(A) = A$. It follows that $c_\mu(i_\mu(c_\mu(A)) \subseteq c_\mu(A) \subseteq A \subseteq U$. Thus, $c_\mu(i_\mu(c_\mu(A)) \subseteq U$. Hence A is μ-$\hat{\beta}g$-closed in X.

Theorem 2.23 *If A is a μ-$\hat{\beta}g$-closed subset in X such that $A \subseteq B \subseteq c_\mu(A)$, then B is an μ-$\hat{\beta}g$-closed in X.*

Proof: Let A be a μ-$\hat{\beta}g$-closed set in X such that $A \subseteq B \subseteq c_\mu(A)$. Let U be a μ-open set in X such that $B \subset U$. Then $A \subset$ U. Since A is μ-$\hat{\beta}g$-closed, $c_\mu(i_\mu(c_\mu(A)) \subseteq U$. Also, $B \subseteq c_\mu(A)$. Then $c_\mu(B) \subseteq c_\mu(c_\mu(A)) = c_\mu(A)$. Now,

$$c_\mu(B) \subseteq c_\mu(A)$$
$$i_\mu(c_\mu(B)) \subseteq i_\mu(c_\mu(A))$$
$$c_\mu(i_\mu(c_\mu(B))) \subseteq c_\mu(i_\mu(c_\mu(A))) \subseteq U$$
$$c_\mu(i_\mu(c_\mu(B))) \subseteq U$$

Hence, B is μ-$\hat{\beta}g$-closed set in X. Therefore, if A is an μ-$\hat{\beta}g$-closed subset in X such that $A \subseteq B \subseteq c_\mu(A)$, then B is an μ-$\hat{\beta}g$-closed in X.

Remark 2.24 *The converse of the above theorem need not be true as seen from the following example.*

Example 2.25 Let $X = \{a, b, c, d\}$. Consider the generalized topology $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the μ-closed sets in X are X, $\emptyset$, $\{b, c, d\}$, $\{a, c, d\}$, $\{c, d\}$ and $\{d\}$. The μ-$\hat{\beta}g$-closed sets in X are $\emptyset$, X, $\{c\}$, $\{d\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, c, d\}$ and $\{b, c, d\}$. Note that the set $\{b\}$ is not μ-$\hat{\beta}g$-closed set in X and the set $\{b, d\}$ is a μ-$\hat{\beta}g$-closed set in X. Observe that the set $\{b\} \subseteq \{b, d\} \subseteq c_\mu(\{b\}) = \{b, c, d\}$.

3. μ-$\hat{\beta}g$ CLOSURE AND μ-$\hat{\beta}g$ INTERIOR IN GTS

We will now define and give some properties of the μ-$\hat{\beta}g$-closure and μ-$\hat{\beta}g$-interior of a set in GTS. Note that these definitions are in parallel to

Definition 3.1 Let (X, μ) be a GTS and $A \subseteq X$.

(i.) The μ-$\hat{\beta}g$-*closure* of A, denoted by $\hat{\beta}gc_\mu(A)$, is the intersection of all μ-$\hat{\beta}g$-closed sets containing A.

(ii.) The μ-$\hat{\beta}g$-*interior* of A, denoted by $\hat{\beta}gi_\mu(A)$, is the union of all μ-βg-open sets contained in A.

Remark 3.2 *Let (X, μ) be a GTS. For any $A \subseteq X$, $A \subseteq \hat{\beta}gc_\mu(A)$ and $\hat{\beta}gi_\mu(A) \subseteq A$.*

Example 3.3 To illustrate Definition 3.1, consider Example 2.2. Note that the μ-closed sets in X are X, $\emptyset$, $\{b,c\}$, $\{a,b\}$, and $\{b\}$. Then, μ-$\hat{\beta}g$-closed sets are $\emptyset$, X, $\{b\}$, $\{a,b\}$ and $\{b,c\}$. Also μ-$\hat{\beta}g$-open sets are X, $\emptyset$ $\{a,c\}$, $\{c\}$ and $\{a\}$. Observe that $A \subseteq \hat{\beta}gc_\mu(A)$ and $\hat{\beta}gi_\mu(A) \subseteq A$.

The next theorem investigates some properties of $\hat{\beta}gc_\mu(A)$ and βg $i_\mu(A)$.

Theorem 3.4 *Let (X, μ) be a GTS and A, B and F be subsets of X.*

(i.) *If A is μ-$\hat{\beta}g$-closed, then $A = \hat{\beta}gc_\mu(A) = \hat{\beta}gc_\mu(\hat{\beta}gc_\mu(A))$;*
(ii.) *$y \in \hat{\beta}gc_\mu(A)$ if and only if for every μ-$\hat{\beta}g$-open set U with $y \in U$, $U \cap A \neq \emptyset$;*
(iii.) *$\hat{\beta}gc_\mu(A) \subseteq c_\mu(A)$;*
(iv.) *If $A \subseteq B$, then $\hat{\beta}gc_\mu(A) \subseteq \hat{\beta}gc_\mu(B)$;*
(v.) *$\hat{\beta}gc_\mu(A) \subseteq \hat{\beta}gc_\mu(\hat{\beta}gc_\mu(A))$; and*
(vi.) *$\hat{\beta}gc_\mu(A) \cup \hat{\beta}gc_\mu(B) \subseteq \hat{\beta}gc_\mu(A \cup B)$.*

Proof: (i.) By Remark 3.2, $A \subseteq \hat{\beta}gc_\mu(A)$. Since A is μ-$\hat{\beta}g$-closed, by Definition 3.1 (i.), $\hat{\beta}gc_\mu(A) \subseteq A$. Hence, $A = \hat{\beta}gc_\mu(A)$. It follows that

$$\hat{\beta} gc_{\mu}(A) = \hat{\beta} g\ c_{\mu}(\hat{\beta} gc_{\mu}(A)).$$

(ii.) Let $y \in \hat{\beta} gc_{\mu}(A)$ and U be a μ-$\hat{\beta} g$-open set with $y \in U$. Suppose that $U \cap A = \emptyset$. Then $A \subseteq X \setminus U$, where $X \setminus U$ is μ-$\hat{\beta} g$-closed. Since $y \notin X \setminus U$, $y \notin \hat{\beta} gc_{\mu}(A)$. Hence, we have a contradiction since $y \in \hat{\beta} gc_{\mu}(A)$. Therefore, $U \cap A \neq \emptyset$.

Conversely, suppose that $y \notin \hat{\beta} gc_{\mu}(A)$. Then there exists a μ-$\hat{\beta} g$-closed set $F \supseteq A$ with $y \notin F$. Hence, $y \in X \setminus F$. Since F is μ-$\hat{\beta} g$-closed, $X \setminus F$ is μ-$\hat{\beta} g$-open and $(X \setminus F) \cap A = \emptyset$ which is a contradiction to the assumption. Therefore, $y \in \hat{\beta} gc_{\mu}(A)$.

(iii.) Let $x \in \hat{\beta} gc_{\mu}(A)$ and let O be any μ-open set with $x \in O$. Then by Corollary 2.16, O is μ-$\hat{\beta} g$-open. Since $x \in \hat{\beta} gc_{\mu}(A)$, by (ii.), $O \cap A \neq \emptyset$. Hence, $x \in c_{\mu}(A)$.

(iv.) Let $x \in \hat{\beta} gc_{\mu}(A)$ and U be a μ-$\hat{\beta} g$-open set with $x \in U$. By (ii.), $U \cap A \neq \emptyset$. Since $A \subseteq B$, it follows that $U \cap B \neq \emptyset$. Therefore, $x \in \hat{\beta} gc_{\mu}(B)$ showing that $\hat{\beta} gc_{\mu}(A) \subseteq \hat{\beta} gc_{\mu}(B)$.

(v.) Let $x \in \hat{\beta} gc_{\mu}(A)$ and let F be any μ-$\hat{\beta} g$-closed set such that $\hat{\beta} gc_{\mu}(A) \subseteq F$. Thus, $x \in F$. By Definition 3.1 (i.), $x \in \hat{\beta} gc_{\mu}(\hat{\beta} gc_{\mu}(A))$. Therefore, $\hat{\beta} gc_{\mu}(A) \subseteq \hat{\beta} gc_{\mu}(\hat{\beta} gc_{\mu}(A))$.

(vi.) Since A and B are contained in $A \cup B$ by (iv.), if follows that $\hat{\beta} gc_{\mu}(A) \subseteq \hat{\beta} gc_{\mu}(A \cup B)$ and $\hat{\beta} gc_{\mu}(B) \subseteq \hat{\beta} gc_{\mu}(A \cup B)$. Therefore, $\hat{\beta} gc_{\mu}(A) \cup \hat{\beta} gc_{\mu}(B) \subseteq \hat{\beta} gc_{\mu}(A \cup B)$.

□

Theorem 3.5 *Let (X, μ) be a GTS and A, B and F be subsets of X.*

(i.) *If A is μ-$\hat{\beta} g$-open, then* $A = \hat{\beta} gi_{\mu}(A) = \hat{\beta} gi_{\mu}(\hat{\beta} gi(A))$;
(ii.) $x \in \hat{\beta} gi_{\mu}(A)$ *if and only if there exists a μ-$\hat{\beta} g$-open set U with* $x \in U \subseteq A$;
(iii.) $i_{\mu}(A) \subseteq \hat{\beta} gi_{\mu}(A)$, *and;*
(iv.) *If* $A \subseteq B$, *then* $\hat{\beta} gi_{\mu}(A) \subseteq \hat{\beta} gi_{\mu}(B)$.

Proof: (i.) Let A be a μ-$\hat{\beta} g$-open subset of X. Since $\hat{\beta} gi_{\mu}(A) \subseteq A$, it suffices to show that $A \subseteq \hat{\beta} gi_{\mu}(A)$. Suppose $x \notin \hat{\beta} gi_{\mu}(A)$. Then by Definition 3.1 (ii.), $x \notin O$ for any μ-$\hat{\beta} g$-open set $O \subseteq A$. Hence, in particular, $x \notin A$ since A is μ-$\hat{\beta} g$-open. Thus, $A \subseteq \hat{\beta} gi_{\mu}(A)$. Therefore, $A = \hat{\beta} gi_{\mu}(A)$. Consequently, $\hat{\beta} gi_{\mu}(A) = \hat{\beta} gi_{\mu}(\hat{\beta} gi_{\mu}(A))$

(ii.) Let $x \in \hat{\beta} gi_{\mu}(A)$. Then $x \in U$ for some μ-$\hat{\beta} g$-open set U with $U \subseteq A$. The converse also follows from Definition 3.1 (ii.).

(iii.) Let $x \in i_{\mu}(A)$. Then there exists a μ-open set O with $x \in O$ and $O \subseteq A$. By Corollary 2.16, O is a μ-$\hat{\beta} g$-open set. Thus there exists a μ-$\hat{\beta} g$-open set O with $x \in O$ and $O \subseteq A$. Hence, by (ii.), $x \in \hat{\beta} gi_{\mu}(A)$.

(iv.) Let $A \subseteq B$. Suppose $x \in \hat{\beta}gi_{\mu}(A)$. By (ii.), there exists μ-$\hat{\beta}g$-open set U with $x \in U \subseteq A$. Since $A \subseteq B$, there exists a μ-$\hat{\beta}g$-open set U with $x \in U \subseteq B$. Thus, $x \in \hat{\beta}gi_{\mu}(B)$. Therefore, $\hat{\beta}gi_{\mu}(A) \subseteq \hat{\beta}gi_{\mu}(B)$.

□

Theorem 3.6 *Let $A \subseteq X$. Then $\hat{\beta}gi_{\mu}(A) = X \setminus [\, \hat{\beta}gc_{\mu}(X \setminus A)]$.*

Proof: Suppose $x \in \hat{\beta}gi_{\mu}(A)$. Then there exists a μ-$\hat{\beta}g$-open set U with $x \in U \subseteq A$. Hence, there exists a μ-$\hat{\beta}g$-closed set $X \setminus U$ with $x \notin X \setminus U \supseteq X \setminus A$. This implies that $x \notin \hat{\beta}gc_{\mu}(X \setminus A)$. Hence, $x \in X \setminus \hat{\beta}gc_{\mu}(X \setminus A)$. Thus, $\hat{\beta}gi_{\mu}(A) \subseteq X \setminus [\hat{\beta}gc_{\mu}(X \setminus A)]$.

Conversely, let $x \in X \setminus \hat{\beta}gc_{\mu}(X \setminus A)$. Then $x \notin \hat{\beta}gc_{\mu}(X \setminus A)$. This implies that there exists a μ-$\hat{\beta}g$-closed set F containing $X \setminus A$ such that $x \notin F$. Hence, there exists a μ-$\hat{\beta}g$-open set $X \setminus F$ with $x \in X \setminus F \subseteq A$. It follows that $x \in \hat{\beta}gi_{\mu}(A)$. Therefore, $\hat{\beta}gi_{\mu}(A) \supseteq X \setminus [\hat{\beta}gc_{\mu}(X \setminus A)]$. Consequently, $\hat{\beta}gi_{\mu}(A) = X \setminus [\hat{\beta}gc_{\mu}(X \setminus A)]$.

□

The next corollary follows immediately from Theorem 3.6.

Corollary 3.7 *Let $A \subseteq X$. Then $\hat{\beta}gc_{\mu}(A) = X \setminus [\hat{\beta}gi_{\mu}(X \setminus A)]$.*

REFERENCES

[1] N. Levine, *Generalized Closed Sets in Topology*, Rend. Circ. Mat. Palermo, 19 (2) (1970), 89-96.

[2] K. Kannan and N. Nagaveni, *On β-Generalized Closed Sets and Open Sets in Topological Spaces*, International Journal of Mathematical Analysis, Vol. 6, 2012, no.57, 2819-2828.

[3] Császár, Á., *Generalized Topology*, *Generalized Continuity*, Acta Mathematica Hungaria 96 (2002), 351-357.

[4] Dugundji, J., *Topology*, New Delhi Prentice Hall of India Private Ltd., 1975.

[5] M.Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 374 – 481.

[6] Császár, Á., *Generalized Open Sets in Generalized Topologies*, Acta Mathematica Hungaria 106 (2005), 53-56.

[7] Orge, K., *Some Forms of Generalized Closed Sets in Generalized Topologies*, Thesis, Mindanao State University-Iligan Institute of Technology, March 2012.

[8] N. Levine, *Generalized Closed Sets in Topology*, Rend. Circ. Mat. Palermo, 19 (2) (1970), 82-88.

[9] Lipschutz, S., Ph. D., *Schaum's Outline of Theory and Problems of General Topology,* McGraw-Hill Incorporated, United States, 1965.

[10] Tampos, M.L., *Alpha Generalized Closed Sets in Generalized and Bigeneralized Topological Spaces*, Thesis, Bohol Island State University – Main Campus, March 2016

[11] Baculta, J. J. *Regular Generalized Star b-sets in Generalized, Bigeneralized and Generalized Fuzzy Topological Spaces,* Dissertation. Mindanao State University – Iligan Insitutute of Technology, May 2015.

[12] Császár, Á., *Generalized Open Sets in Generalized Topologies*, Acta Mathematica Hungaria 106 (1-2) (2002), 351-357.

[13] Levine, N., *Generalized Closed Sets in Topology*, Rend. Circ. Mat. Palermo, 19 (1982), 82-88, 89-96.

[14] Császár, Á., *Generalized Open Sets in Generalized Topologies*, Acta Mathematica Hungaria 120 (2008), 275-279.

[15] Njastad, O., *On Some Classes of Nearly Open Sets*, Pacific Journal Math, 15 (1965), 961-970.

[16] Barbe M. R. Stadler and Peter F. Stadler, *Generalized Topological Spaces in Evolutionary Theory and Combinatorial Chemistry.* (2001)

[17] Wright, S., *The Roles of Mutation, Inbreeding, Crossbreeding and Selection in Evolution.* In: Jones, D. F., ed., Int. Proceedings of the Sixth International Congress on Genetics. Vol.1, (1932) 356-366.

[18] Palaniappan N and Rao KC (1993) *Regular generalized closed sets,* Kyunpook Math. J33: 211-219.

[19] Benchalli S.S., Wali R.S., *On rw-Closed Sets in Topological Spaces.* Bulliten of the Malaysian Mathematical Sciences Society. (2)30(2)(2007), 99-110.

[20] Duangphui, T., Boonpok, C., Viriyapong C., *Continuous Functions on Bigeneralized Topological Spaces*. Int. Journal of Math. Analysis Vol.5,2011, no.24, 1165-1174.