Poisson Regression models to reduce Waiting Time for Hospital Service. Case Study: Nakhonpathom Hospital Thailand.

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Abstract

The growing population has led to increase in waiting time and overcrowding in the hospital service, a Poisson regression model has been developed to analyze the time series of count data. In finding a Poisson regression model, parameters are estimated and goodness-of-fit is utilized to carefully extract the best model to fit the count data. The marginal effect is the basis function which can be used in the Poisson regression model. This study attempted to analyze actual operations of a hospital and proposed modifications in the system to reduce waiting times for the patients, which should lead to an improved view of the quality of service provided. To develop a Poisson regression analysis model for the above situation, we need to define a model for the expected number of patients for hospital services cases. Here, two underlying variables are of interest, "waiting time" and "hospital services". Since "waiting time" have been categorized seven groups. The variable "hospital services" which contains four categorizes (No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and 30 baht for welfare health service (Gold cards (30W)). As a result, significant levels of causal variables are not expected to be identical for each model. We find that 30 baht for welfare health service (Gold cards (30W)) category has a higher rate of increase in the average waiting time. The marginal effect is a basis function that can be used in the Poisson regression. It allows into arrive at better predictions of hospital service and rehabilitation decision making.

Keywords: *Poisson regression model, Risk Rate model, Log-linear model, Queueing model, Hospital service*

1 Introduction

In the categorical data analysis literatures (Heien, 2004), the survival models and/or the Poisson regression models are treated differently than standard logit models. In general, these models are termed as Rate Models or Risk Rate Models. In its simplest form, a rate is defined as the number of individuals or observations possessing a particular characteristic divided by the total amount of exposure to the risk of having such a characteristic. The Poisson regression Models can easily be connected to the standard Poisson Models (Daniel and Xie, 2000). Then the Poisson Models are directly related to the Exponential Models by making conversion of rates per unit interval with the waiting time until the first occurrence. Here we use a Poisson regression Model to determine the likelihood of the demand for hospital services in a queueing system is to identify factors associated with increased health care utilization; particularly those factors related to hospital services. This is a difficult task for several reasons.

From queuing theory standpoint, a welfare hospital department can be viewed as a system of queues and different types of servers. A quantitative analysis of the wait time problem in welfare hospital department is dependent upon the identification of a methodology which recognizes the structure of the problem as that of a queuing system. Two modes of analysis are generally suggested by the structure of this type of problem: queuing models and discrete event simulations.

The Admitting department consists of four major areas: Front desk, Registration desk, Waiting area, and Financial Consulting area (within Business Department). When patient enters the Admitting department, they are asked by front-desk clerk to provide name and reason for visit. Admitting clerk determines patient's type (No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and the 30 baht for welfare health service (Gold cards (30W)) and create new account using Hospital Informational System. Admitting serves most outpatient and inpatient types, with an exception for: No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and the 30 baht for welfare health service (Gold cards (30W)) It is essential to assess the relationship between hospital services and average waiting time in a queuing system. The clerk also clarifies if patient was pre-registered for this service or not. If the answer is yes, the clerk gets patient's documentation ready for the admission representative. Then the patient receives an assigned number and is asked to wait in admitting waiting area for admitting representative to call the number. Admitting representative determines if the patient ever receives the service at the hospital and if so, pull up patient's data from Meditech and verifies patient's personal information. If the patient is visiting the hospital for the first time, admitting clerk creates patient's profile in the Hospital Information Database system.

Law and Kelton (2001) proposed an algorithm of a successful computer simulation study. This algorithm includes the following key steps: 1. Problem formulation, 2. Data collection and the conceptual model design, 3. The validation of the model, 4. The constructions of the computer representation of the model, 5. The verification of the model, 6. The design of experiments needed to address the problem, 7. Production runs using the computer model, 8. The statistical analysis of the data obtained from the production runs, and 9. The interpretation of the results.

First, even in the case of constant demand levels over the day, statistical fluctuations in individual patient waiting times and the variability in the time needed by a provider to service patients can create long delays even when overall average steady state capacity is greater than average demand. Second, the magnitude of delays is a log-linear function of the demand for hospital services level, and are thus impossible to predict without the use of a queueing model. (Green and Nguyen 2001).

2 Methodology of Risk Rate Models

2.1 Risk Rate Models analysis

Let *Y*i be the number of patients of an event of interest for the ith subject and denote the independent variables by x_i, i= 1,…,n. We assume that Yi follows a Poisson distribution, Yi ~ Poisson (λ_i), with density

$$f(y_i|x_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!} \quad (1)$$

Let t1 , t2,…,tn be the waiting times of the nth individual, and assume the distribution function to be F(t) = Pr(T < t) with probability density function f(t). The risk rate is denoted by, and can be viewed the instantaneous probability of an event in the interval [t, t+1], given the event has not occurred before time t. formally, the risk rate (Charles and McFadden 1981), is defined by the following limit:

$$\mu(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} Pr[t \le T < t + \Delta t \,|\, T \ge t] \quad (2)$$

The density of an exponential distribution with parameter μ is given by

$$f(t) = \mu e^{-\mu t} \text{ ,} t>0. \quad (3)$$

The distribution function equal

$$F(t) = 1 - \mu e^{-\mu t}, \quad t \ge 0. \quad (4)$$

For this distribution, we have

$$E(x) = \frac{1}{\mu}, \quad \sigma^2(x) = \frac{1}{\mu^2}, \quad (5)$$

The probability of an event not occurring up to time *t* is given by the function

$$P(t) = Pr[T > t] = 1 - F(t) = e^{-\mu t} \quad (6)$$

Assuming the waiting times are exponentially distributed, the equation (6) may be written as:

$$P(t) = e^{(-\mu t_i)} \quad (7)$$

The risk rate is defined by the ratio. $\mu(t_i) = \frac{f(t)}{P(t)} = \frac{f(t)}{1-F(t)} = \frac{\mu e^{-\mu t}}{e^{-\mu t}} = \mu$ (8)

The general risk rate model may be written as: $\mu(x_i^T \beta_i) = e^{(\beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n)}$ (9)

where $x^T = [1, x_1, x_2, \ldots, x_n]$, and $\beta_0, \beta_1, \ldots, \beta_n$ are unknown constants as the rate is determined by several regressors. This exponential risk rate model can be estimated using a Poisson regression Models for counts. In a time interval of length *t*, the probability of *y* events is given by: $Pr(y \,|\, \mu, t) = \frac{(\mu t)^y e^{(-\mu t)}}{y!}$ (10)

Because the mean number of events in the time interval *t*is $\lambda = \mu t$, for the i^{th} individual, the expected number of events in the time interval t_i is

$$\lambda_i = \mu_i t_i, \quad \text{or} \quad \lambda_i = t_i e^{(x_i^T \beta_i)} \quad (11)$$

Taking the log of the Poisson means results in the log-linear regression model:

$$\frac{\lambda_i}{t_i} = e^{(x_i^T \beta_i)} \quad \text{or} \quad log(\frac{\lambda_i}{t_i}) = x_i^T \beta_i \quad (12)$$

Namely, $log(\lambda_i) - log(t_i) = x_i^T \beta_i$, (13)

2.2. Goodness-of-Fit

The log-likelihood function of the process cannot be the only index of fit because the likelihood-ratio-statistics is dependent on the size of the sample. Different values of the log-likelihood function result when competing models, namely models that differ in the number of parameters, are fitted to the same data. The number of parameters, in general, should be more than one, and significantly less than the number of observations. To assess the model goodness-of-fit, we need to know how one model fits relative to another. An indicator of a model goodness-of-fit that measures the extent to which the current model deviates from a more generalized model is given by the likelihood-ratio-statistics:

$$G^2 = -2\log\left(\frac{L_c}{L_f}\right) = -2(\log L_c - \log L_f), \tag{14}$$

where $\log L_c$ is the log-likelihood of the current model, and $\log L_f$ is the log-likelihood of the more generalized model. The likelihood ratio statistics has a Chi-Square distribution with $K_2 - K_1$ degrees of freedom, where K_2 and K_1 denote the number of parameters in the more generalized model and the current model, respectively (McCullagh and Nelder 1989).

2.3. Marginal Effects

For Poisson regression, the marginal effects can be thought of as the relative risk associated with a certain variable. The overall mean effect in (15) is

$$\lambda(x^T\beta) = e^{(x^T\beta)} \tag{15}$$

Then, the marginal effect due to the $k^{\underline{th}}$ factor can be considered as

$$\hat{\theta}_k = \bar{x}_k e^{(\bar{x}^T\hat{\beta})} \tag{16}$$

where $\bar{x}_k$ is the mean of the $k^{\underline{th}}$ factor values in the sample and $\bar{\mathbf{x}}^T$ is the vector of the means of the factor values in the sample. An estimate of θ_k can be computed as

$$\theta_k = e^{(\beta)} \tag{17}$$

3 Estimation results and interpretation

We assume that victims arrive at the first hospital at moment *t*, and the inter-arrival time is exponential and that the arrival rate is $\lambda(t)$; there are $s(t)$ parallel servers in the hospital, the service time is exponential and that the service rate is $\mu(t)$; and the hospital can accommodate the largest number of victims is *N*.

Setting the inter-arrival time as, and there is only one event (arrival or completing treatment) at most occur in $\Delta(t)$. If there are K victims at moment t in the system, and:

1) The arrival probability $\lambda(t)\,\Delta(t)$ is when only one victim arrives at the hospital during $\Delta(t)$.;

2) The arrival probability $o(\Delta t)$; is when more than one victims arrive at the hospital during $\Delta(t)$;

3) Assuming that the arrival time between victims is independent during Δt the service probability completed a victim is $\min(s(t),K)\mu(t)\Delta t$ in Δt;

4) The service probability completed more than one victim is $o(\Delta t)$; in Δt.

If the arrival victims follow the first three assumptions, they will be subject to the non-homogeneous Poisson process. The value of Δt can be small enough to reduce the error of calculating the transient probability.

We use $p_i(t)$ to denote the probability of i victims in the system at the moment t based on the initial system. Therefore, $P_0(0) = 1$ if $i > 0$, $P_i(0) = 0$. Then according to the following equation, we can calculate the probability of i victims in the system.

This result is graphically depicted in Fig. 1, is the M/M/1 and the M/M/s queue, which shows the probability of n outpatients arriving in the system (P_n) slowly decreases as the number n outpatients arriving in the system (P_n) increases and similarly Poisson distribution. Suppose we observe a non-homogeneous Poisson process $X(t)$ with rate $\lambda(t)$ by Poisson regression analysis. Another categorical class of log-linear models that are commonly considered is that in which $\log(\lambda(t))$ is assumed to be a polynomial with unknown coefficients.

The M/M/1queue and The M/M/s queue of Probability of n outpatients arriving in system

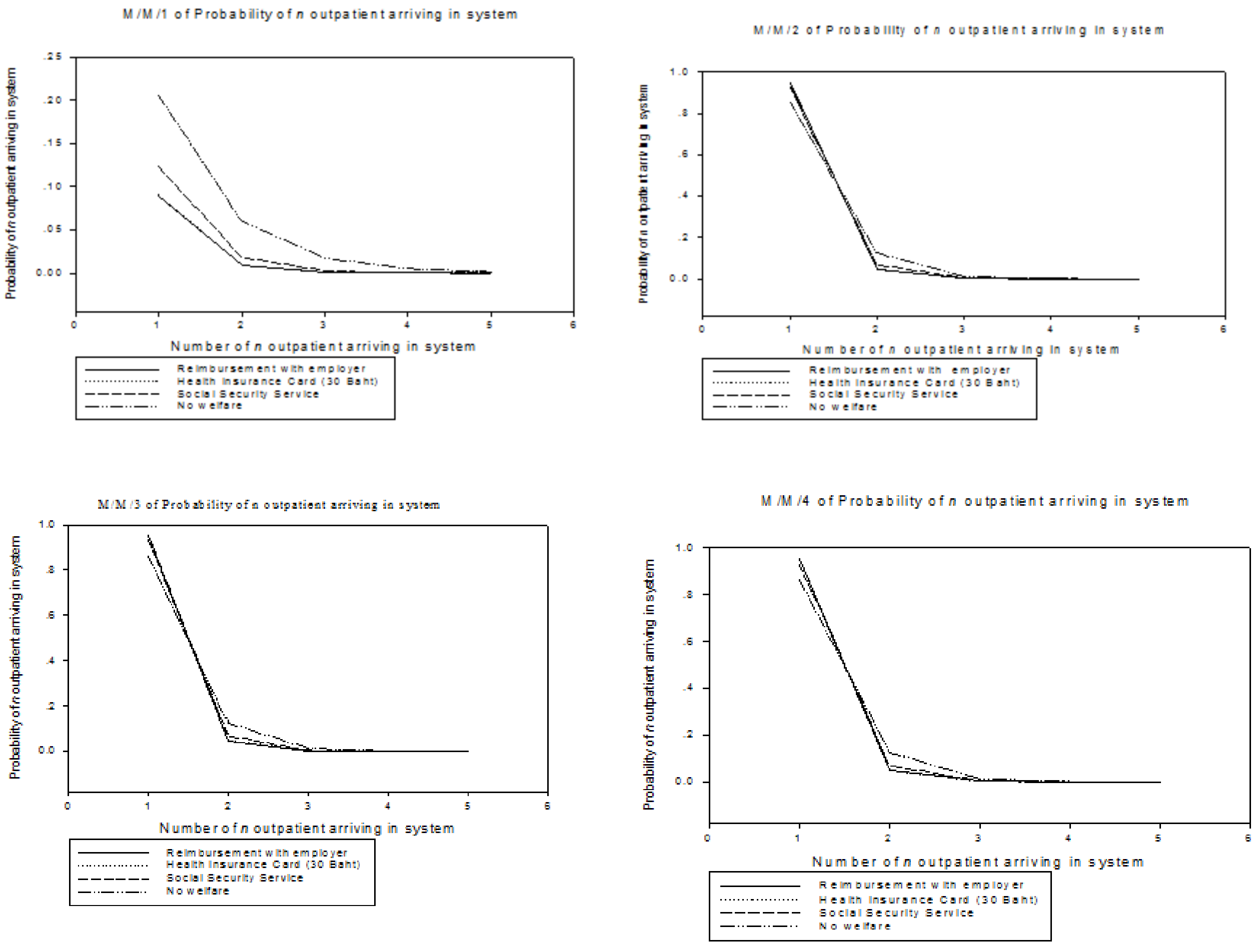

Fig 1. We find that the probability of n outpatients arriving in the system slowly decreases as the number n outpatients arriving in the system increases and similarly Poisson distribution.

The data set was obtained quantitative data from a survey by the Nakhonpathom Hospital in Thailand for the year 2017 in Fig.1 above was used to estimate the probability of *n* outpatients arriving in the system and number of *n* outpatients arriving in the system. The M/M/1 queue of probability of *n* outpatients arriving in the system (P_n) have Reimbursement to employer (RE), the 30 baht for welfare health service (Gold cards (30W)), Social Security Service (SS) and No welfare (NW) respectively. The M/M/1 queue of probability of *n* outpatients arriving in the system (P_n) slowly decreases as the number *n* of outpatients arriving in the system increases, similar to the Poisson distribution The M/M/s queue of probability of *n* outpatients arriving in the system (P_n) have Reimbursement to employer (RE), the 30 baht for welfare health service (Gold cards (30W)), Social Security Service (SS) and No welfare (NW) respectively. The M/M/s queue of probability of *n* outpatients arriving in the system (P_n) slowly decreases as the number *n* of outpatients arriving in the system increases, similar to the Poisson distribution

The following data was obtained quantitative data from a survey by the Nakhonpathom Hospital in Thailand for the year 2017. The Admitting Department is one of the most highly congested hospital services, and faces a great deal of pressure, compared with other components of the health care system. Admitting clerk determines patient's type (No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and the 30 baht for welfare health service (Gold cards (30W)) and create new account using Hospital Informational System. Admitting serves most outpatient and inpatient types, with an exception for: No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and the 30 baht for welfare health service (Gold cards (30W)). It is essential to assess the relationship between hospital services and average waiting time in a queuing system. Using SAS to perform the iterations necessary for the maximum likelihood method, the following results have been obtained.

Table 1 Contingency Table of Outpatients

		Hospital Services				Total
Average waiting time		**NW** (h_1)	**RE** (h_2)	**SSS** (h_3)	**30W** (h_4)	
1-20	(t_1)			1	3	4
21-40	(t_2)		6	7	33	46
41-60	(t_3)	10	12	9	31	62
61-80	(t_4)		8		1	9
81-100	(t_5)	33	7	5		45
101-120	(t_6)				30	30
> 120	(t_7)	4				4
Total		47	33	22	98	200

To develop a Poison regression Models model for the above situation, we need to define a model for the expected number of patients for welfare cases, $E\ (Y_{ij})$ in terms of the variables of interest. Here, two underlying variables are of interest, "waiting time" and "welfare". Since "waiting time" has been categorized into seven groups, we will use six dummy variables to index them. The variable "welfare" which contains four categorizes, requires only three dummy variables. Thus, one possible model for the expected number of patient for welfare cases in the $(i,j)^{th}$ group can be written as:

$E(Y_{ij}) = \mu_{ij} = \eta_{ij}\lambda_{ij}$, where $log\lambda_{ij} = \alpha + \beta E, i = 1, 2, ..., n, j = 1, 2, ..., m$.

Using this model, we can write the risks λ_{Ij} in terms of the parameters α_i and β to obtain

$log\lambda_{i0} = \alpha + \alpha_i$ and $log\lambda_{i1} = \alpha + \alpha_i + \beta$,

since $log\lambda_{i1} - log\lambda_{i0} = (\alpha + \alpha_i + \beta - \alpha - \alpha_i)$

$log\lambda_{i1} - log\lambda_{i0} = \beta$, so $\theta_k = e^{(\beta)}$

4 Model Results

The data set in Table 1 above was used to estimate the average waiting time and hospital services for medical and health services on the Poisson regression analysis with hospital services variables characterized by the marginal effect ($\theta_k = e^{(\beta)}$). Two separate models were specified and estimated for each state since the various causal factors vary over time, as a result, significant level of causal variables are not expected to be identical for each model. The model is to compare the average waiting time (t) and hospital services (h). We also computed a chi-square test relation log-linear model and Poisson regression Models:

Saturated log-linear model: $\log\lambda = \mu + \mu_i t_i + \mu_j h_j + \mu_{ij} t_i h_j$

(18)

Poisson regression Models: $\log\lambda_{ij} = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3ij}$ $i=1,2,...,n; j=1,2,..,m$

(19)

where are dummy variables and the interaction variable is, and the variable x_1 corresponds to the No welfare (NW) case, x_2 corresponds to the Reimbursement to employer (RE) case, x_3 corresponds to the Social Security Service (SS) case, x_4 corresponds to the 30 baht for welfare health service (Gold cards (30W)) case, x_5 corresponds to the group with average waiting time 1-20 min, x_6 corresponds to the group with average waiting time 21-40 min, x_7 corresponds to the group with average waiting time 41-60 min, x_8 corresponds to the group with average waiting time 61-80 min, x_9 corresponds to the group of with average waiting time 81-100 min, x_{10} corresponds to the group with average waiting time 101-120 min, x_{11}corresponds to the group with average waiting time > 120 min, and $x_{12} = x_1 x_5$, $x_{13} = x_2 x_5$, $x_{14} = x_3 x_5$,…,$x_{39} = x_4 x_{11}$.

A saturated log-linear model rate defined. We use the deviations between the maximized log-likelihood from each model to perform a series of Chi-square tests in order to ascertain which model gives the best fit.

So, the saturated log-linear model is $log(\lambda) = \mu + \sum_{i=1}^{n} \mu_i t_i + \sum_{j=1}^{m} \mu_i h_j + \sum_{i=1}^{n}\sum_{j=1}^{m} \mu_{ij} t_i h_j$

(20)

The main effects model of factors t and w is $log(\lambda) = \mu + \sum_{i=1}^{n} \mu_i t_i + \sum_{j=1}^{m} \mu_i h_j$

(21)

The main effects model of factor *t* is $log(\lambda) = \mu + \sum_{i=1}^{n} \mu_i t_i$

(22)

The main effects model of factor *w* is $log(\lambda) = \mu + \sum_{j=1}^{m} \mu_i h_j$

(23) Poisson regression Models:

So, the saturated log-linear model is $log(\lambda) = \beta_0 + \sum_{i=1}^{28} \beta_i x_i$

(24)

The main effects model of factors *t* and *w* is $log(\lambda) = \beta_0 + \sum_{i=1}^{11} \beta_i x_i$

(25)

The main effects model of factor *t* is $log(\lambda) = \beta_0 + \sum_{i=1}^{7} \beta_i x_i$

(26)

The main effects model of factor *w* is $log(\lambda) = \beta_0 + \sum_{i=1}^{4} \beta_i x_i$

(27)

The following results were obtained.

The saturated log-linear model yields

$log(\lambda)$ = -14.8949+16.2841h_1-0.6423h_2 -1.1326h_3+16.1394t_1+18.406t_2+18.8536t_3+17.6167t_4+18.10t_5+ 18.2963t_6-32h_1 t_1-16 h_2t_1-0.0365h_3t_1-32h_1t_2-0.0718 h_2t_2-0.2821h_3t_2-15.9804h_1t_3-0.5691h_2t_3-0.4642h_3t_3-32h_1t_4-16h_3t_4-16h_1t_5+32h_1t_6-16h_2t_6-16h_3t_6, (28)

log-likelihood = 383.3958, df = 26

The main effects model of factors *w* and *t* yields

$log(\lambda)$=0.9728+0.04h_1-0.962h_2-1.559h_3-0.0957t_1+2.588t_2+2.9627t_3+1.6526t_4+2.5138t_5+2.4172t_6 , (29)

log-likelihood = 375.2952,Chi-square=10.857, df = 17

The main effects model of factor yields

$log(\lambda)$=0.05539-0.1605t_1+2.481t_2+2.7188t_3+1.2t_4 +12.4825 t_5+2.8036 t_6, (30)

log-likelihood = 341.413,Chi-square=80.413,df=20

The main effects model of factor *h* yields

$log(\lambda)$=3.2908-0.4153h_1-1.1057h_2-1.6614h_3, (31)

log-likelihood = 307.565,Chi-square=125.923,df =23

Goodness-of-Fit

Using these results, we tested the competing models using the likelihood-ratio statistics as described in section 2.2 in order to determine in goodness of fit. To perform the tests, we started by testing the saturated model in (30) against the main factors model in (31), and then tested the main factors model against its nested counterparts. The results of Chi-square tests, performed with α = 0.05, are as follows:

Table 2.Test goodness-of-fit model of hospital services

Test	G^2	Df
(29) vs (28)	-2[(375.2952)-(383.3958)]=16.2012	9
(30) vs (29)	-2[(341.4130)-(375.2952)]=67.7644	3
(31) vs (29)	-2[(307.5654)-(375.2952)]=135.4596	3

The main factors model in (29) compared to saturated model (with all the interactions) in (28) has adequate fit model. The model in (29) has adequate fit compared to all models. The main factors of welfare model in (31) compared to all models in (29). The model in (31) has adequate fit compared to welfare model. Thus, we decided to choose the main factors model in (31) as the adequate model for this data set**.**

The main effects model; $log(\lambda)= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$

$$log(\lambda) = 3.2908-0.4153w_1-1.1057\ w_2-1.6614w_3-0.00001w_4 \qquad (32)$$

Table 3. Mean marginal effect of hospital services

Variable		Marginal effect
No welfare (NW),	(x_1)	$e^{-0.4153} = 0.66014$
Reimbursement to employer (RE)	(x_2)	$e^{-1.1057} = 0.33098$
Social Security Service (SS)	(x_3)	$e^{-1.6614} = 0.18987$
The 30 baht for welfare health service (Gold cards (30W))	(x_4)	$e^{-0.00001} = 0.99999$

Tables 2 and 3 shows that the parameter estimates of hospital services are significant at the 5% level. The results indicate into arrive at better predictions of health services. As for the hospital services class, the categories are No welfare (NW), Reimbursement to employer (RE), Social Security Service (SS) and the 30 baht for welfare health service (Gold cards (30W)). The marginal effects are computed as described in Section 2.3. The marginal effect for the first factor, The No welfare (NW), group, is calculated as $\hat{\theta}_1 = \exp(\hat{\beta}_1)$ = 0.66014. This means, the target population of the No welfare (NW) group has a 0.66014 times. The marginal effect for the second factor, The Reimbursement to employer (RE) group, is calculated as $\hat{\theta}_2 = \exp(\hat{\beta}_2)$= 0.33098. This means the target population of the Reimbursement to employer (RE) group has a 0.33098 times. The marginal effect for the third factor, The Social Security Service (SS) group, is calculated as $\hat{\theta}_3 = \exp(\hat{\beta}_3)$ = 0.18987. This means the target population of Social Security Service (SS) group has a 0.18987 times. The marginal effect for the fourth factor, The 30 baht for welfare health service (Gold cards (30W)) group, is calculated as $\hat{\theta}_4 = \exp\left(\hat{\beta}_4\right)$ = 0.99999. This means the target population of the30 baht for welfare health service (30W) group has a 0.99999 times.

This results in a monotonic increase in the waiting time rate function. This result is graphically depicted in Fig. 2, which shows a clear increase in the waiting time rate function for categorical welfare variables.

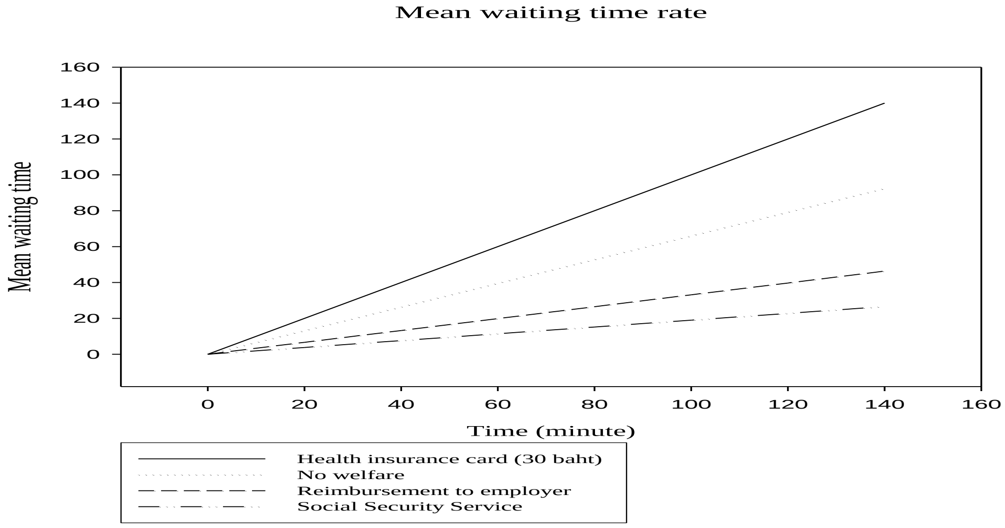

Fig. 2. The waiting time rate function is linearly increasing for each of the categorical welfare variables.

As a result, significant levels of causal variables are not expected to be identical for each model. We find Fig.2 that 30 baht for welfare health service (Gold cards (30W)) category has a higher rate of increase in the average waiting time. The marginal effect is a basis function that can be used in the Poisson regression. It allows into arrive at better predictions of hospital service and rehabilitation decision making.

5 Conclusions

This paper has surveyed the use of queuing theory for the analysis of different types of waiting time and welfare hospital. Models for estimating waiting time and welfare hospital, models for system design, and models for evaluating appointment systems have been presented. The survey has reviewed models for departments (or units), facilities, and systems. We find that the 30 baht for welfare health service (Gold cards (30W)) category has a higher rate of increase in the average waiting time. The marginal effect ($\hat{\theta}_k$) is a basis function that can be used in the Poisson regression analysis for flexibility. In this paper, we have described an easily implemental estimation procedure for the coefficients of hospital services in the queuing system. The estimation approach is based on using a series of prediction probabilities. Furthermore, the methodology for determining the prediction probabilities from the waiting time model is developed. Finally, testing for statistical significance was carried out and the risk rate function was found to be increasing.

Waiting time in the welfare hospital can be reduced through implementation of quantitative methods, understanding of best practices, and commitment to change. For instance, queuing models of welfare hospital department activity have a broad range of potential applications. One of the most promising areas is the study of welfare hospital overcrowding. A critical capability afforded by patient flow simulation is the reconstruction of the factors that are responsible for overcrowding. This allows a more detailed understanding of the relationship between the observed conditions and related outcomes that could lead to informed optimization decisions.

As long as increasing the productivity of healthcare organizations remains important, analysts will seek to apply relevant models to improve the performance of healthcare processes. This paper shows that many models are available today. However, analysts will increasingly need to consider the ways in which distinct queuing systems within an organization interact. Developing appropriate models of the links (or interfaces) between the distinct queuing systems is an important direction for future research.

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