GALERKIN WEIGHTED RESIDUAL METHOD FOR MAGNETO-HYDRODYNAMIC(MHD) MIXED CONVECTION FLOW IN A VERTICAL CHANNEL FILLED WITH POROUS MEDIA

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Abstract

This research paper focuses on the problem of magneto-hydrodynamic mixed convections in a porous medium embedded in a vertical channel with a vertical axial variation of temperature in the upward direction along the wall. We provide a Galerkin Weighted Residual Method (GWRM) to the solution of the coupled ordinary differential equations of momentum and heat transfer. Results obtained were analyzed using tables and graphs and the effect of the thermophysical parameters arising from the flow were studied and it is seen to be in good agreement with Literature.

Keywords: weighted residual methods, porous media, MHD.

Introduction:

The porous media heat transfer problems have numerous thermal engineering applications such as geothermal energy recovery, crude oil extraction, thermal insulation, ground water pollution, oil extraction, thermal energy storage, thermal insulations, and flow through filtering devices. Hamad and Bashir[1].

Due to the wide applications of flow through porous media several researches and scientist has devoted much studies to the phenomenon of flow through both vertical and horizontal channels, examples include the work of Mishra et al[2] who studied the mixed convention flow in a porous medium bounded by two vertical channels. They presented an analytical study by converting the second order ordinary differential equation of momentum into a fourth order differential equation thereby unable to provide a solution to the equation of heat transfer but neglected the effect of magnetic field.

Srivastava and Singh[3] also studied mixed convection in a composite system bounded by vertical walls with a vertical axial variation of temperature in the upward direction along the walls. They also presented an analytical study by converting the second order ordinary differential equations into a fourth order differential equation and thereby unable to provide graphical solution to the equation of heat transfer and also neglected the effect of magnetic field.
Nobari[4] carried out a numerical study of the mixed convective flow in a vertical channel by applying finite difference method based on projection algorithm. While the mixed convective flow between vertical parallel plates has been solved numerically by Guillet et al[5]. Okedayo et al[6] carried out an analytical study of Viscous Dissipation Effect on Flow through a Horizontal Porous Channel with Constant Wall Temperature and a Periodic Pressure Gradient. Similarly Oyelami and Dada[7] transient magnetohydrodynamic flow of Eyring-Powell fluid in a porous medium. Furthermore Kavitha et al[8] studied the Convective MHD Flow of A Second Order Fluid in an Inclined Porous Channel Considering Darcy's Effect while Ojha and Panda [9] Free Convection MHD flow between two vertical walls filled with a Porous matrix.

The subject of Magneto hydrodynamics has attracted the attention of a large number of scientists due to its diverse application. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter and radio propagation through the ionosphere. In engineering, it finds its application in MHD pumps and MHD bearing. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes, and ionized gases. At the high temperature attained in some engineering devices, gas, for example, can become ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic.
Das *et al.[10]* analyzed Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Panda *et al* [11], investigated Heat
and mass transfer in MHD flow of viscous fluid past a vertical plate under oscillatory suction velocity with heat source. Sahoo *et al.[12]* analyzed the MHD mixed convection stagnation point flow and heat transfer in a porous medium.
The weighted Residual method is a class of approximate methods that is capable of producing simple analytical solutions. These methods include the colocation, Galerkin's method, least

square and orthogonal colocation methods. Several authors have devoted their attention to the application of these methods to the solutions of equations arising from both Newtonian and Non-Newtonian fluids. Examples of such include the work of Hatami et al[13] and shaqin and Huoyuan[14]

Being motivated by the above studies especially the works of Mishra et al[2] and Srivastava and Singh[3] who did not provide the solutions of the equations of heat transfer we undertake to provide a Galerkin weighted residual solution for the mixed convection flow in a vertical channel filled with porous media thereby providing analytical and graphical solutions to both the momentum and heat transfer problem.

Problem formulation:

Consider the steady fully developed laminar free convection flow between two vertical walls filled with an electrically conducting fluid saturated porous medium under a constant pressure gradient. The walls are separated by a distance $2L$ apart and having a linear axial temperature variation. The $x'-axis$ is taken along the vertical direction while $y'-axis$ perpendicular to it. For fully developed laminar flow, the velocity has only the vertical component and is a function of y' only. As a result of these assumptions, the equation of motion in $x'-direction$ and energy equations are given as follows

$$0 = -\frac{\partial p}{\partial x'} - \lambda_0[1-\beta(T'-T'_0)] + \mu_{eff}\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_f}{K}u' - \frac{\sigma\beta_0^2 u'}{\lambda_0} \qquad (1)$$

$$u'\frac{\partial T'}{\partial x'} = \alpha\frac{\partial^2 T'}{\partial y'^2} \qquad (2)$$

$$u' = 0, \quad T' = T'_0 + Nx', \quad at \quad y' = L;$$
$$u' = 0, \quad T' = T'_0 + Nx', \quad at \quad y' = -L \qquad (3)$$

Where Da, g, K, L, N , p' Ra , Rb , T' , To , u' , u , x', y' , β , μ_{eff} , μ_f , λ_0 $\lambda, \theta, \alpha, \sigma, \beta$ are the Darcy number,Acceleration due to gravity, Permeability of the porous medium, half channel width, constant pressure gradient of the fluid, Rayleigh number, Ratio of the effective viscosity of the porous domain to that of the viscosity of the fluid, Temperature of the fluid, Temperature in a reference form, velocity of the fluid in a non-dimensional form, co-ordinate along the vertical direction, co-ordinate along the horizontal direction, co-efficient of thermal expansion, effective viscosity of the fluid saturated porous medium, viscosity of the fluid, density in a reference state, density of the fluid, temperature in a non-dimensional form, Thermal diffusivity respectively, electrical conductivity and magnetic inductor.

Introducing the following dimensionless quantities into the momentum and energy equations

$$\left.\begin{aligned} & y=\frac{y'}{L},\ u=\frac{u'L}{\alpha P_x},\ \theta=\frac{T-T_0-Nx}{NLP_x} \\ & P_x=\frac{\lambda L^3}{\alpha\mu_f}(-(\frac{1}{\lambda_0\partial x}+g)+g\beta Nx') \end{aligned}\right\} \quad (4)$$

We have

$$\left.\begin{aligned} & R_b\frac{\partial^2 u}{\partial y^2}-\frac{u}{D_a}+R_a\theta-Mu=-1 \\ & and \\ & \frac{\partial^2\theta}{\partial y^2}-u=0 \\ & with \\ & u=0,\ \theta=0,\ at\ \ y=1 \\ & u=0, \theta=0\ \ at\ \ y=-1 \end{aligned}\right\}$$

(5)

Where,

$$R_b=\frac{\mu_{eff}}{\mu_f},\ D_a=\frac{K}{L^3},\ R_a=\frac{\lambda g N\beta L^4}{\alpha\mu_f}, M=\frac{L^2\sigma\beta_0^2}{\lambda\mu_f}$$

(6)

Where R_b is the ratio of effective viscosity of the porous domain to that of the viscosity of the fluid, R_a is the Rayleigh's number and D_a is the Darcy number, M is the magnetic parameter.

Problem Solution

We proceed to solve the momentum and heat transfer equations by the Galerkin weighted residual method using the following algorithm.

Consider a differential operator *L* acting on a function *u* to produce a function *f*.

$$L[u(y)]=f(y) \quad (7)$$

The objective is to approximate $u(y)$ by a function $\bar{u}(y)$, which is a linear combination of basis functions chosen from a linearly independent set of the form

$$\bar{u}(y) = \sum_{i=0}^{n} a_n \phi_n(y) \qquad (8)$$

Substituting this into the governing equation we obtain the residual equation of the type

$$R(c_i, r) = L\left[\sum_{i=0}^{n} a_n \phi_n(y)\right] - f(y) \qquad (9)$$

It is required that the residuals be orthogonal to a set of weight functions and in this case to the chosen basis functions $\phi_n(y)$. Which results into a system of algebraic equations, which are then solved for the coefficients $a_n's$. Here we select the following basis function of the form

$$u(y) = a_0(1-y^2) + a_1(y-y^3) + a_2(1-y^4)$$

$$\theta(y) = b_0(y^2-1) + b_1(y^3-y) + b_2(y^4-1)$$

For the momentum and heat transfer equations respectively and satisfies the given set of boundary conditions.

Results and Discussion:

Examples of some analytical results obtained for different values of the various thermophysical parameters are: when $R_a = 10, R_b = 1.0, D_a = 0.1, M = 10$ we have

$$u(y) = \text{-}0.03208189812(1-y^2) \text{ - } 0.02844559428y(1-y^2) + 0.07897919881(1-y^4)$$

$$\theta(y) = \text{-}0.02188769820(y^2-1) \text{ - } 0.001415507618y(y^2-1) + 0.003136521835(y^4-1)$$

And when $R_a = 10, R_b = 1.0, D_a = 0.1, M = 15$ we obtain

$$u(y) = \text{-}0.03545354050(y^2-1) \text{ - } 0.02790741442y(y^2-1) + 0.07486454059(y^4-1)$$

$$\theta(y) = \text{-}0.01824167025(y^2-1) \text{ - } 0.001111435566y(y^2-1) + 0.002448406897(y^4-1)$$

The impact of the various thermophysical parameters are depicted in Figure.1-8. Figure.1 shows the velocity profile for various values of the magnetic field parameter (M), while the ratio of the effective viscosity, Raleigh's number and Darcy number are kept constant. It can be seen that the velocity decreases as the magnetic field parameter increases which shows that the applied magnetic field opposes the fluid motion. While in Figure.2, shows the velocity profile for variation in the Darcy number (Da), it is observed that the velocity increases as Darcy number increases. In Figure.3 the velocity profile for various values of the ratio of the effective viscosity(Rb) is shown It is observed that the velocity decreases as Ratio of effective viscosity(Rb) increases this is apparent due to the higher influence of the viscous force on the velocity field. While in Figure.4 we depicts the influence of the Raleigh's number on the velocity profile, it is observed that increase in the Raleigh's number leads to a corresponding decrease in the velocity profile this is due to the dominance of opposing buoyancy forces.

In Figure.5 the effect of the magnetic field parameter is shown which shows that as the magnetic field parameter increases the temperature decreases since the motion of the fluid is slowed down by the magnetic field. And in Figure.6 the effect of the Darcy number is displayed and it is evident that higher values of the Darcy number leads to proportional increase in the temperature profile and this could be the effect of higher velocity values as the Darcy number increases.

Figure.7 shows the temperature profile for the variation in the Ratio of effective viscosity to viscosity of fluid (Rb). It shows that temperature decreases as the Ratio of effective viscosity (Rb) increases. Figure.8 shows the temperature profile for variation in the Rayleigh's number(Ra), it is seen that temperature decreases as the Rayleigh's number increases.

Table.1 shows the skin friction coefficients and Nusselt numbers for some choice of the thermophysical constants when the magnetic field parameter is absent, it is noticeable that both the skin friction coefficient and Nusselt numbers decreases with increase in the parameters shown, these values were however compared with values obtained from the Runge-Kutta method and it is observed that the maximum error in both the skin fiction coefficient and Nusselt number is 2% which shows that the GWRM is quite efficient and accurate.

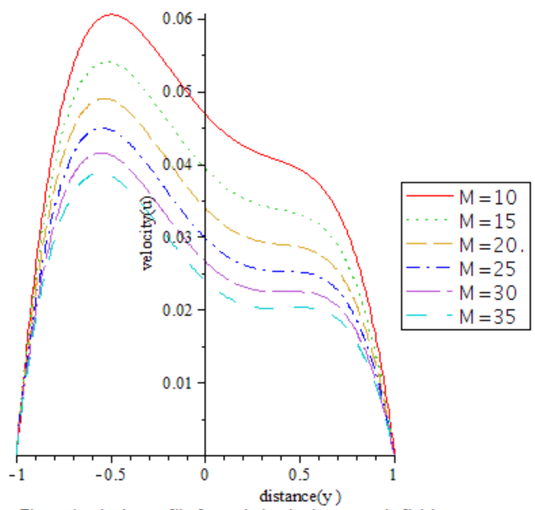

Figure.1 velocity profile for variation in the magnetic field parameter

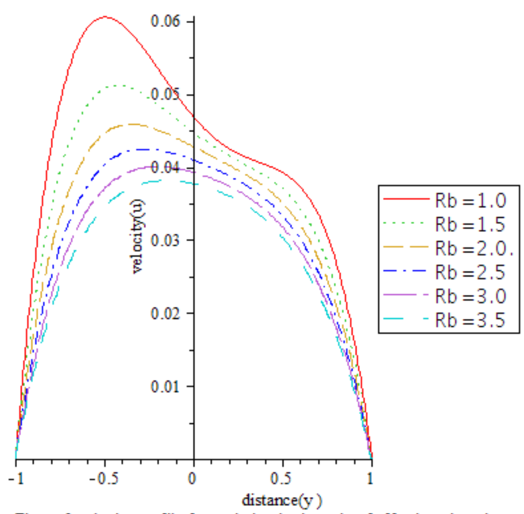

Figure.3 velocity profile for variation in the ratio of effective viscosity

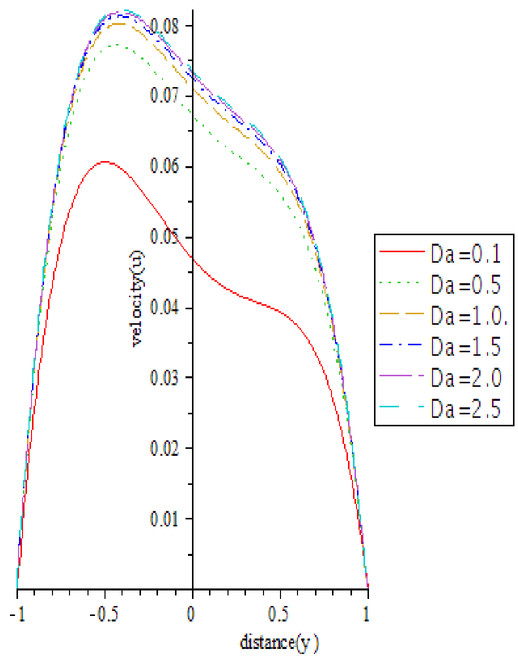

Figure.2 velocity profile for variation in the Darcy Number

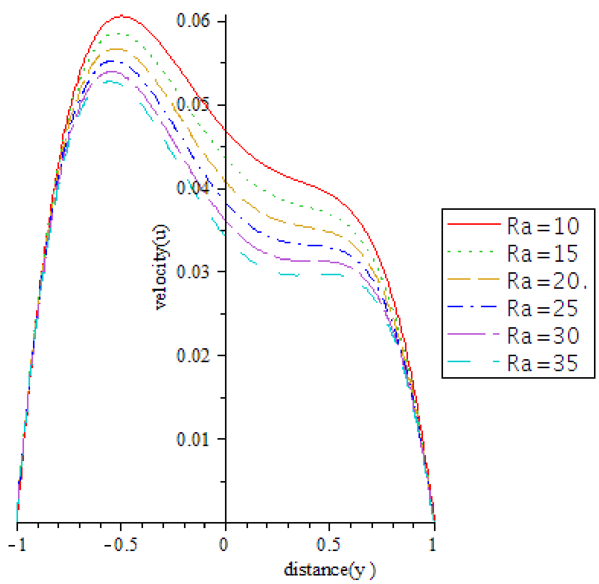

Figure.4 velocity profile for variation in the Raleigh Number

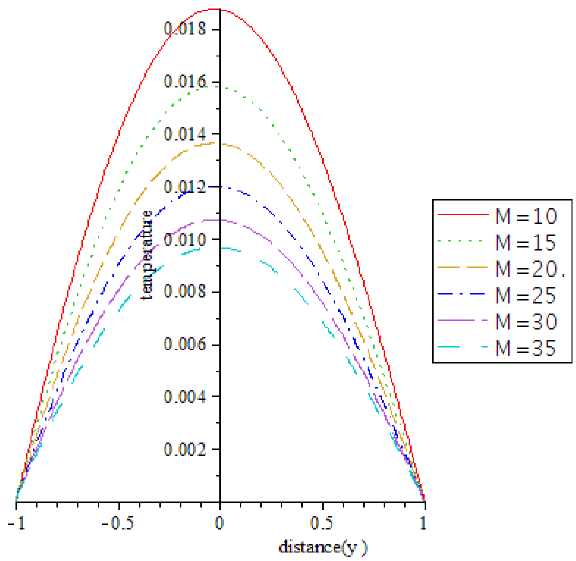

Figure.5 temperature profile for variation in the magnetic field parameter

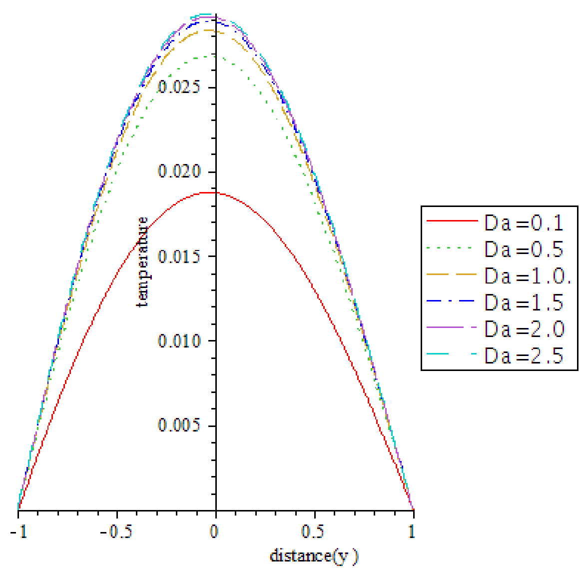

Figure.6 temperature profile for variation in the Darcy Number

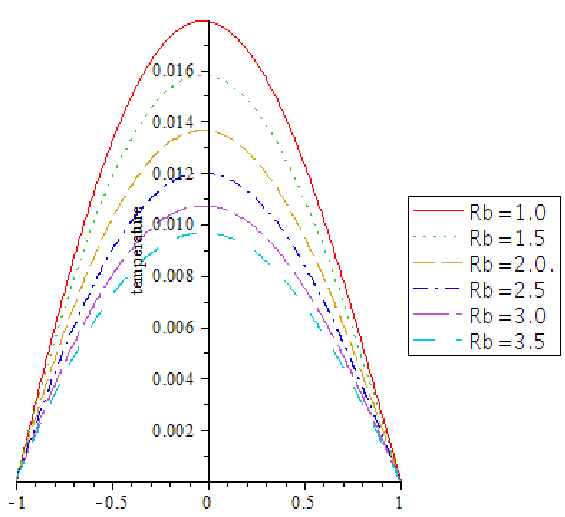

Figure.7 temperature profile for variation in the ratio of effective viscosity

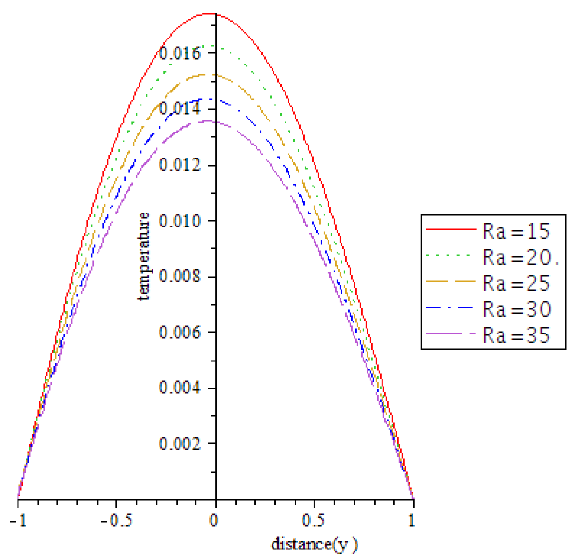

Figure.8 temperature profile for variation in the Raleigh Number

Table.1: Skin Friction Coefficients and Nusselt Number.

R_a	R_b	Da	GWRM		RUNGE-KUTTA		% error in skin-friction	% error in Nusselt number
			Skin-friction	Nusselt number	Skin-friction	Nusselt number		
10	1.0	0.1	0.26902895	0.05331743	0.27480939	0.05250076	2	1.6
10	1.5	0.1	0.22047383	0.04855178	0.22142767	0.04818352	0.4	0.8
10	2.0	0.1	0.19011869	0.04479824	0.18964882	0.04469033	0.2	0.2
15	1.0	0.1	0.25533507	0.04807185	0.26112462	0.04712042	2	2
15	1.5	0.1	0.20883224	0.04398347	0.20963463	0.04349546	0.4	1.1

Summary and Conclusion.

The problem of MHD Mixed Convection Flow in a Vertical Channel using the Galerkin Weighted Residual Method (GWRM) has been studied. The coupled equation of momentum and Energy has been solved and the flow and energy transfer has been analyzed using both tables and graphs.

1. The maximum velocity and temperature occurs at the centre of the channel except for variation in the Raleigh's Number.
2. Increase in Rayleigh number leads to a decrease in velocity.
3. Increase in the ratio of the effective viscosity leads to a decrease in the velocity.
4. Increase in the Darcy number leads to an increase in the velocity.
5. The temperature decreases with the increase in the ratio of effective viscosity.

6. Increase in Darcy number leads to increase in Temperature.
7. Increase in the magnetic field parameter leads to a decrease in both velocity and temperature profiles.

References

1. M.A.A. Hamad and M. A. Bashir Similarity Solutions of the Effect of Variable Viscosity on Unsteady Mixed Convection Boundary Layer Flow over a Vertical Surface Embedded in a Porous Medium via HAMAD Formulations World Applied Sciences Journal(2011) 12 (4): 519-530
2. Mishra A. K., Paul T. and Singh A. K. (2002) Mixed convection flow in a porous medium bounded by two vertical walls. *Forschung im Ingenieurwesen* 67,198-205.
3. Srivastava N. and Singh A. K. 2010 Mixed Convection in a Composite System Bounded by Vertical Walls ***J**ournal of **A**pplied **F**luid **M**echanics*, Vol. 3, No. 2, pp. 65-75.
4. Nobari, M.R.H. (2007). A numerical study of mixed convection in a vertical channel flow impinging on a horizontal surface. *International Journal of Thermal Sciences* 46, 989-997
5. Guillet, C., T. Mare and Nguye C.T. (2007). Application of a non-linear local analysis method for the problem of mixed convection instability. *International Journal of Non-Linear Mechanics* 42, 981-988.
6. Okedayo. T. Gideon, Owoloko E. A. and Osafile O. E.(*2011*) Viscous Dissipation Effect on Flow through a Horizontal Porous Channel with Constant Wall Temperature and a Periodic Pressure Gradient *International Journal of Advances in Science and Technology, Vol. 3, No.6*
7. Oyelami, F.H. and Dada, M.S . (2016) transient magnetohydrodynamic flow of eyring-powell fluid in a porous medium *Ife Journal of Science* vol. 18, no. 2
8. Kavitha K.R., Ramana Murthy Ch. V. and Reddy A. Rami (*2011*) Convective MHD Flow of A Second Order Fluid in An Inclined Porous Channel Considering Darcy's Effect *International Journal of Advances in Science and Technology, Vol. 3, No.1*8.
9. Ojha, J. P. and Panda N.C. (2015) Free Convection MHD flow between two vertical walls filled with a Porous matrix. Ultra Scientist Vol. 27(2)A, 155-158
10. Das S.S., Satapathy A., Das J.K. and Panda J.P. (2009), *Int. J. Heat Mass Transfer.* Vol. 52, 5962-5969.
11. Panda J.P., Dash N. and Dash G.C.,(2010) *Modelling, Measurement and Control,B,*AMSE, Vol.*79,* No.1-2, pp.52-65.
12. Sahoo S.N., Panda J.P. and Dash G.C(2013)., *Proc.Natl. Acad. Sci., India*, Sect. A Phys. Sci., Vol.83(4), pp. 371-381.
13. Hatami M., J Hatami and D. D. Ganji(2014) computer simulation of MHD blood conveying gold nanoparticles as a third grade non-Newtonian nanofluid in a hollow porous vessel. computer methods and programs in biomedicine 113, 632-641.
14. G. Shoqin and D. Huoyuan(2008) Negative least squares methods for the incompressible magneto-hydrodynamic equations, Acta Maths.Sci. B28(3) 675-684.