

## SUPER $\delta(\delta g)^*-e$ -CLOSED SETS and SUPER $\delta(\delta g)^*-e$ -CONTINUOUS FUNCTIONS

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**Abstract:** In this paper, we introduce and investigate the new class of generalized closed sets called super  $\delta(\delta g)^*-e$ -closed set in topological space using  $\delta$ -generalized  $e$ -closed sets. Moreover we analyze the relations between super  $\delta(\delta g)^*-e$ -closed sets and various closed sets. We also investigate new class functions named super  $\delta(\delta g)^*-e$ -continuous, and study its fundamental properties and compare it with some other types of functions.

**Key words:**  $\delta$ - $e$ -closed, super  $g$ - $e$ -closed sets, super  $\delta g$ - $e$ -closed sets, super  $\delta g^*$ - $e$ -closed sets, super  $\delta(\delta g)^*-e$ -closed sets, super  $\delta(\delta g)^*-e$ -continuous.

**1.Introduction:** The concept of generalized closed (briefly,  $g$ -closed) sets were introduced and investigated by Norman Levine [6] in 1970. Velicko[10] introduced  $\delta$ -open sets which are stronger than open sets in 1968. By combining the concepts of  $\delta$ -closedness and  $g$ -closedness, Julian Dontchev [1] proposed a class of generalized closed sets called  $\delta g$ -closed set in 1996. A.P. Dhana Balan, C.Santhi[4] introduced and investigated the new concept of  $\delta$ generalized  $e$ -closed sets. In one sense this paper is a continuation of our paper [4]. The aim of this paper is to introduce a new class of generalized closed sets called super  $\delta(\delta g)^*-e$ -closed. In section 2, we provide some preliminaries of topological spaces which are used to carry out our work. In section 3, we introduce the definition of super  $\delta(\delta g)^*-e$ -closed sets and analyse the relations between super  $\delta(\delta g)^*-e$ -closed sets and  $\delta$ - $e$ -closed, super  $\delta g^*$ - $e$ -closed, super  $g\delta$ - $e$ -closed, super  $\delta g^\#$ - $e$ -closed, super  $\pi g$ - $e$ -closed sets. In section 4, we introduce super  $\delta(\delta g)^*-e$ -continuous function. The notion of super  $\delta(\delta g)^*-e$ -continuous function between the topological spaces has been defined and this relates such a concept to various functions, which give the composite. The purpose of this paper is to formulate some simple properties of super  $\delta(\delta g)^*-e$ -closed sets and to study, based on this concept, suitable generalizations of the concept of continuous functions, Unless otherwise stated explicitly, topological spaces on which no separation axioms are assumed. The end or omission of proof will be denoted by ■.

**2.Preliminaries:** We start this section with a basic definition on behalf of which we obtain the super  $\delta(\delta g)^*-e$ -closed set.

**Definition 2.1** A subset  $A$  of  $(X, \tau)$  is called a regular open (resp semi open,  $\pi$ -open,  $\delta$ -open) if  $A = \text{int}(cl(A))$  (resp.  $A \subseteq cl(\text{int}(A))$ ), it is the finite union of regular open sets, it is the union of regular open sets). The  $\delta$ -interior [10] of a set  $A$  of  $X$  is the union of all regular open sets of  $X$  contained in  $A$  and is denoted by  $\delta\text{-int}(A)$ . The subset  $A$  is called  $\delta$ -open if  $A = \text{int}_\delta(A)$ . The complement of  $\delta$ -open set is called  $\delta$ -closed  $\subseteq cl(\delta\text{-int}(A))$ , ie a set  $A \subset X$  is  $\delta$ -closed if  $A = cl_\delta(A)$ , where  $cl_\delta(A) = \{x \in X : \text{int}(cl(U)) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$ . A subset  $A$  of  $X$  is called  $e$ -open [5] (resp.  $e$ -closed) if  $A \subseteq cl(\delta\text{-int}(A)) \cup \text{int}(\delta\text{-cl}(A))$  (resp.  $cl(\delta\text{-int}(A)) \cap \text{int}(\delta\text{-cl}(A)) \subseteq A$ ). The intersection of all  $e$ -closed sets containing  $A$  in  $X$  is called the  $e$ -closure of  $A$  and is denoted by  $e\text{-cl}(A)$ . The union of all  $e$ -open sets contained in  $A$  in  $X$  is called the  $e$ -interior of  $A$  and is denoted by  $e\text{-int}(A)$ . The collection of all  $e$ -open sets of  $X$  is denoted by  $eo(X)$  and the collection of all  $e$ -closed sets of  $X$  is denoted by  $ec(X)$ . A subset  $A$  of  $X$  is  $e$ -regular open [7] if it is  $e$ -open and  $e$ -closed and is denoted by  $e$ - $r$ -open. A subset  $A$  of  $X$  is  $\pi$ - $e$ -open if it is the finite union of  $e$ - $r$ -open sets. The  $\delta$ - $e$ -interior of a subset  $A$  of  $X$  is the union of all  $e$ - $r$ -open sets of  $X$  contained in  $A$  and is denoted by  $e\text{-int}_\delta(A)$ . The subset  $A$  is called  $\delta$ - $e$ -open if  $A = e\text{-int}_\delta(A)$ . ie, a set is  $\delta$ - $e$ -open if it is the union of  $e$ - $r$ -open sets.

The complement of  $\delta$ - $e$ -open set is called  $\delta$ - $e$ -closed. ie,  $A$  is called  $\delta$ - $e$ -closed if  $A = e-cl_{\delta}(A)$  where  $e-cl_{\delta}(A) = \{x \in X / A \cap int(cl(U)) \neq \emptyset, \text{ for every } U \in \tau \text{ and } x \in U \text{ and } A \in ec(X)\}$ .

**Definition 2.2** A subset  $A$  of a topological space  $(X, \tau)$  is called

1. *generalized closed[6](briefly  $g$ -closed)* if  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
2.  *$\delta$ generalized closed[1](briefly  $\delta g$ -closed)* if  $\delta cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
3. *generalized  $\delta$ -closed[2](briefly  $g\delta$ -closed)* if  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta$ -open in  $(X, \tau)$ .
4. *regular generalized closed[8](briefly  $rg$ -closed)* if  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is regular open in  $X$ .
5.  *$\delta g^*$ -closed [9]* if  $\delta cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $g$ -open in  $(X, \tau)$ .
6.  *$\delta g^{\#}$ -closed [11]* if  $\delta cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta$ -open in  $(X, \tau)$ .
7.  *$\pi g$ -closed [3]* if  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\pi$ -open in  $(X, \tau)$ .

**Definition 2.3** A subset  $A$  of a topological space  $(X, \tau)$  is called

1. *generalized  $e$ -closed[5](briefly  $g$ - $e$ -closed)* if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
2.  *$\delta$ generalized  $e$ -closed[4](briefly  $\delta g$ - $e$ -closed)* if  $e-cl_{\delta}(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
3.  *$g\delta$ - $e$ -closed* if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta$ -open in  $(X, \tau)$ .
4.  *$rg$ - $e$ -closed* if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is regular open in  $(X, \tau)$ .
5.  *$\delta g^*$ - $e$ -closed* if  $e-cl_{\delta}(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $g$ -open in  $(X, \tau)$ .
6.  *$\delta g^{\#}$ - $e$ -closed* if  $e-cl_{\delta}(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta$ -open in  $(X, \tau)$ .
7.  *$\pi g$ - $e$ -closed* if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\pi$ -open in  $(X, \tau)$ .
8.  *$\hat{g}$ - $e$ -closed* if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is semi open in  $(X, \tau)$ .

The complements of the above mentioned sets are called their respective open sets.

**Remark 2.4.** [4] Note that none of the following implications is reversible.

$e$ - $r$ -closed(open)  $\rightarrow$   $\pi$ - $e$ -closed(open)  $\rightarrow$   $\delta$ - $e$ -closed(open)  $\rightarrow$   $\delta g$ - $e$ -closed(open)  $\rightarrow$   $g$ - $e$ -closed(open).

### 3. Super $\delta(\delta g)^*$ - $e$ -closed sets

In this section we introduce and investigate new class of sets namely  $\delta(\delta g)^*$ - $e$ -closed set and super  $\delta(\delta g)^*$ - $e$ -closed sets.

**Definition 3.1** A subset  $A$  of a topological space  $(X, \tau)$  is said to be

1. *Super  $\delta(\delta g)^*$ - $e$ -closed set* if  $e-cl_{\delta}(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ .

The class of all super  $\delta(\delta g)^*$ - $e$ -closed sets of  $(X, \tau)$  is denoted by  $\mathbb{S}\delta(\delta G)^*eC(X, \tau)$ .

2.  *$\delta(\delta g)^*$ - $e$ -closed* if  $e-cl_{\delta}(A) \subseteq U$  whenever  $A \subseteq U, U$  is  $\delta g$ -open in  $(X, \tau)$ . It is independent of  $\delta g$ - $e$ -closed sets and weaker than  $\delta g^*$ - $e$ -closed sets. The class of  $\delta(\delta g)^*$ - $e$ -closed sets is properly placed between the class of  $\delta g^*$ - $e$ -closed set and  $\delta g^{\#}$ - $e$ -closed set and a chain of relations follows easily  $e$ - $r$ -closed  $\rightarrow$   $\pi$ - $e$ -closed  $\rightarrow$   $\delta$ - $e$ -closed  $\rightarrow$   $\delta g^*$ - $e$ -closed  $\rightarrow$   $\delta(\delta g)^*$ - $e$ -closed  $\rightarrow$   $\delta g^{\#}$ - $e$ -closed  $\rightarrow$   $g\delta$ - $e$ -closed

**Example 3.2** Let  $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ . Then  $\{a, b\} \subseteq X$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $(X, \tau)$ .

**Theorem 3.3** Finite union of super  $\delta(\delta g)^*$ - $e$ -closed sets is always a  $\delta$ -generalized  $e$ -closed set.

Proof: Let  $\{A_i / i=1, 2, \dots, n\}$  be a finite class of super  $\delta(\delta g)^*$ - $e$ -closed subsets of a space  $(X, \tau)$ . Then for each  $\delta g$ - $e$ -open set  $U_i$  in  $X$  containing  $A_i, e-cl_{\delta}(A_i) \subseteq U_i, i \in \{1, 2, \dots, n\}$ . Hence  $\cup_i A_i \subseteq \cup_i U_i = V$ . Since finite union of  $\delta g$ - $e$ -open sets in  $(X, \tau)$  is also  $\delta g$ - $e$ -open set in  $(X, \tau), V$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ . Also  $\cup_i e-cl_{\delta}(A_i) = e-cl_{\delta}(\cup_i(A_i)) \subseteq V$ . Therefore  $\cup_i A_i$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $(X, \tau)$ .

**Remark 3.4** Intersection of any two super  $\delta(\delta g)^*$ - $e$ -closed in  $(X, \tau)$  need not be super  $\delta(\delta g)^*$ - $e$ -closed.

**Theorem 3.5** The intersection of a super  $\delta(\delta g)^*$ - $e$ -closed set and a  $\delta$ - $e$ -closed set is always super  $\delta(\delta g)^*$ - $e$ -closed.

Proof: Let  $A$  be super  $\delta(\delta g)^*$ - $e$ -closed and let  $F$  be  $\delta$ - $e$ -closed. If  $U$  is an  $\delta g$ - $e$ -open set with  $A \cap F \subseteq U$ , then  $A \subseteq U \cup F^c$  and so  $e-cl_\delta(A) \subseteq U \cup F^c$ . Now  $e-cl_\delta(A \cap F) \subseteq e-cl_\delta(A) \cap F \subseteq U$ . Hence  $A \cap F$  is super  $\delta(\delta g)^*$ - $e$ -closed.

**Definition 3.6** A subset  $A$  of a topological space  $(X, \tau)$  is said to be

1. Super  $\delta g$ - $e$ -closed set if  $e-cl_\delta(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $e$ -open in  $(X, \tau)$ .
2. Super  $\delta g^*$ - $e$ -closed set if  $e-cl_\delta(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $g$ - $e$ -open in  $(X, \tau)$ .
3. Super  $\delta g^\#$ - $e$ -closed set if  $e-cl_\delta(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $\delta$ - $e$ -open in  $(X, \tau)$ .
4. Super  $g\delta$ - $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $\delta$ - $e$ -open in  $(X, \tau)$ .
5. Super  $\pi g$ - $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $\pi$ - $e$ -open in  $(X, \tau)$ .
6. Super  $g$ - $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $e$ -open in  $(X, \tau)$ .
7. Super  $g^*$ - $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $g$ - $e$ -open in  $(X, \tau)$ .
8. Super  $*g$ - $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $\hat{g}$ - $e$ -open in  $(X, \tau)$ .
9. Super  $\delta\hat{g}$ - $e$ -closed set if  $e-cl_\delta(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $\hat{g}$ - $e$ -open in  $(X, \tau)$ .

**Proposition 3.7** Every  $\delta$ - $e$ -closed set is super  $\delta(\delta g)^*$ - $e$ -closed but not conversely.

Proof: Let  $A$  be a  $\delta$ - $e$ -closed set and  $U$  be any  $\delta g$ - $e$ -open set containing  $A$ . Since  $A$  is  $\delta$ - $e$ -closed,  $e-cl_\delta(A) = A$ . Therefore  $e-cl_\delta(A) = A \subseteq U$  and hence  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed. ■

**Example 3.8** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ . In this topology the subset  $\{a, b\}$  is super  $\delta(\delta g)^*$ - $e$ -closed but not  $\delta$ - $e$ -closed, since  $e-cl_\delta(\{a, b\}) = X \neq \{a, b\}$ .

**Remark 3.9** Every  $\delta$ - $e$ -closed set is  $\delta(\delta g)^*$ - $e$ -closed but not conversely.

**Proposition 3.10** Every super  $\delta g^*$ - $e$ -closed set is super  $\delta(\delta g)^*$ - $e$ -closed but not conversely.

Proof: Let  $A$  be super  $\delta g^*$ - $e$ -closed and let  $U$  be any  $\delta g$ - $e$ -open set containing  $A$  in  $X$ . By result 2.4, every  $\delta g$ - $e$ -open set is  $g$ - $e$ -open and since  $A$  is super  $\delta g^*$ - $e$ -closed,  $e-cl_\delta(A) \subseteq U$ . Hence  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed. ■

**Example 3.11** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, X, \{a, b\}\}$ . Then the subset  $\{b, c\}$  is super  $\delta(\delta g)^*$ - $e$ -closed but not super  $\delta g^*$ - $e$ -closed in  $X$ , since  $e-cl_\delta(\{b, c\}) = X \not\subseteq \{b, c\}$  whenever  $\{b, c\}$  is  $g$ - $e$ -open in  $(X, \tau)$ .

**Remark 3.12** Every super  $\delta g^*$ - $e$ -closed set is  $\delta(\delta g)^*$ - $e$ -closed but not conversely.

**Proposition 3.13** Every super  $\delta(\delta g)^*$ - $e$ -closed set is super  $g\delta$ - $e$ -closed but not conversely.

Proof: Let  $A$  be super  $\delta(\delta g)^*$ - $e$ -closed set and  $U$  be any  $\delta$ - $e$ -open set containing  $A$  in  $X$ . By result 2.4, every  $\delta$ - $e$ -open set is  $\delta g$ - $e$ -open and  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed, we have  $e-cl_\delta(A) \subseteq U$ . For every  $e$ -closed set  $A$  of  $X$ ,  $e-cl(A) \subseteq e-cl_\delta(A)$  and so  $e-cl(A) \subseteq U$  and hence  $A$  is super  $g\delta$ - $e$ -closed. ■

**Example 3.14** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ . Then the subset  $\{a\}$  is super  $g\delta$ - $e$ -closed but not super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , since  $e-cl_\delta(\{a\}) = X \not\subseteq \{a\}$  whenever  $\{a\}$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ .

**Remark 3.15** Every  $\delta(\delta g)^*$ - $e$ -closed set is  $g\delta$ - $e$ -closed but not conversely.

**Proposition 3.16** Every super  $\delta(\delta g)^*$ - $e$ -closed set is super  $\delta g^\#$ - $e$ -closed but not conversely.

Proof: Let  $A$  be super  $\delta(\delta g)^*$ - $e$ -closed set and  $U$  be any  $\delta$ - $e$ -open set containing  $A$  in  $X$ . By result 2.4, every  $\delta$ - $e$ -open set is  $\delta g$ - $e$ -open and  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed, we have  $e-cl_\delta(A) \subseteq U$ . Hence  $A$  is super  $\delta g^\#$ - $e$ -closed. ■

**Example 3.17** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}\}$ . Then the subset  $\{a\}$  is super  $\delta g^\#$ - $e$ -closed but not super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , since  $e-cl_\delta(\{a\}) = X \not\subseteq \{a\}$  whenever  $\{a\}$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ .

**Remark 3.18** Every  $\delta(\delta g)^*$ - $e$ -closed set is  $\delta g^\#$ - $e$ -closed but not conversely.

**Definition 3.19** A subset  $A$  of a topological space  $(X, \tau)$  is said to be super regular generalized- $e$ -closed set if  $e-cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is  $e$ -regular open in  $(X, \tau)$ .

**Proposition 3.20** Every super  $\delta(\delta g)^*$ - $e$ -closed set is super  $rg$ - $e$ -closed, but not conversely.

Proof: Let  $A$  be super  $\delta(\delta g)^*$ - $e$ -closed and  $U$  be any  $e$ -regular open set containing  $A$  in  $X$ . By remark 2.4, every  $e$ -regular open set is  $\delta g$ - $e$ -open and  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed,  $e-cl_\delta(A) \subseteq U$ . For every  $e$ -closed set  $A$  of  $X$ ,  $e-cl(A) \subseteq e-cl_\delta(A)$  and so we have  $e-cl(A) \subseteq U$ . Hence  $A$  is super  $rg$ - $e$ -closed. ■

**Example 3.21** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ . Then the subset  $\{a\}$  is super  $rg$ - $e$ -closed but not super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , since  $e-cl_\delta(\{a\}) = X \not\subseteq \{a\}$  whenever  $\{a\}$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ .

**Remark 3.22** Every  $\delta(\delta g)^*$ - $e$ -closed set is  $rg$ - $e$ -closed but not conversely.

**Proposition 3.23** Every super  $\delta(\delta g)^*$ - $e$ -closed set is super  $\pi g$ - $e$ -closed but not conversely.

Proof: Let  $A$  be super  $\delta(\delta g)^*$ - $e$ -closed and  $U$  be any  $\pi$ - $e$ -open set containing  $A$  in  $X$ . By result 2.4, every  $\pi$ - $e$ -open set is  $\delta g$ - $e$ -open and  $A$  is super  $\delta(\delta g)^*$ - $e$ -closed,  $e-cl_\delta(A) \subseteq U$ . For every  $e$ -closed set  $A$  of  $X$ ,  $e-cl(A) \subseteq e-cl_\delta(A)$  and so we have  $e-cl(A) \subseteq U$  and hence  $A$  is super  $\pi g$ - $e$ -closed. ■

**Example 3.24** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ . Then the subset  $\{c\}$  is super  $\pi g$ - $e$ -closed but not super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , since  $e-cl_\delta(\{c\}) = \{b, c\} \not\subseteq \{c\}$  whenever  $\{c\}$  is  $\delta g$ - $e$ -open in  $(X, \tau)$ .

**Remark 3.25** Every  $\delta(\delta g)^*$ - $e$ -closed set is  $\pi g$ - $e$ -closed but not conversely.

**Example 3.26** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}\}$ . In this topology the subset  $\{b\}$  is  $g$ - $e$ -closed, super  $g$ - $e$ -closed, super  $^*g$ - $e$ -closed, super  $gd$ - $e$ -closed, but not super  $\delta g$ - $e$ -closed, not super  $\delta g^*$ - $e$ -closed, not super  $\delta \hat{g}$ - $e$ -closed, not super  $\delta(\delta g)^*$ - $e$ -closed.

#### 4. Super $\delta(\delta g)^*$ - $e$ -Continuous Functions

In this section, we define super  $\delta(\delta g)^*$ - $e$ -continuous function and study some of their characterizations.

Let  $(X, \tau)$  and  $(Y, \sigma)$  are topological spaces. A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

1.  $g$ -continuous [6] if  $f^{-1}(V)$  is  $g$ -closed in  $X$ , for every closed set  $V$  of  $Y$ .
2.  $g$ - $e$ -continuous [4] if  $f^{-1}(V)$  is  $g$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .

**Definition 4.1** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called super  $\delta(\delta g)^*$ - $e$ -continuous if  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .

**Example 4.2** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a, b\}\}$  and  $Y = \{p, q\}$ ,  $\sigma = \{\emptyset, Y, \{p\}\}$ . By example 3.11,  $\{b, c\}$  is super  $\delta(\delta g)^*$ - $e$ -closed set in  $X$ . Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a function defined as:  $f(a) = p$ ,  $f(b) = f(c) = q$ . Since  $V = \{q\}$  is closed in  $Y$ ,  $f^{-1}(V) = \{b, c\}$  is a super  $\delta(\delta g)^*$ - $e$ -closed set of  $(X, \tau)$ . Thus  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous.

**Theorem 4.3** For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  the following properties are equivalent:

- (i)  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous,
- (ii)  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -open in  $X$ , for every open set  $V$  of  $Y$ .
- (iii)  $f^{-1}(F)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ , for every closed set  $F$  of  $Y$ .

Proof: (i)  $\Rightarrow$  (ii) Let  $V$  be a open set of  $Y$ . Since  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous, there exists  $U \in \mathcal{S}\delta(\delta G)^*eO(X)$  such that  $U = f^{-1}(V)$ . Hence  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -open in  $X$ . (ii)  $\Rightarrow$  (i) Let  $V$  be a open set of  $Y$ . By (ii),  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -open in  $X$ , take  $U = f^{-1}(V)$ . Hence  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous.

(ii)  $\Rightarrow$  (iii) Let  $F$  be any closed set of  $Y$ . Since  $Y \setminus F$  is a open in  $Y$ , by (ii),  $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$  is super  $\delta(\delta g)^*$ - $e$ -open. ie,  $f^{-1}(F)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ .

(iii)  $\Rightarrow$  (ii) Let  $V$  be any open set of  $Y$ . Since  $Y \setminus V$ , say  $F$ , is closed in  $Y$ , by (iii),  $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed. ie,  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -open in  $X$ . ■

**Definition 4.4** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

1.  $\delta$ - $e$ -continuous if  $f^{-1}(V)$  is  $\delta$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
2. Super  $\delta g^*$ - $e$ -continuous if  $f^{-1}(V)$  is super  $\delta g^*$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
3. Super  $rg$ - $e$ -continuous if  $f^{-1}(V)$  is super  $rg$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
4. Super  $g\delta$ - $e$ -continuous if  $f^{-1}(V)$  is super  $g\delta$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
5. Super  $\delta g^\#$ - $e$ -continuous if  $f^{-1}(V)$  is super  $\delta g^\#$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .
6. Super  $\pi g$ - $e$ -continuous if  $f^{-1}(V)$  is super  $\pi g$ - $e$ -closed in  $X$ , for every closed set  $V$  in  $Y$ .

**Theorem 4.4** Every  $\delta$ - $e$ -continuous function is super  $\delta(\delta g)^*$ - $e$ -continuous. but not conversely.

Proof: Let  $f$  be a  $\delta$ - $e$ -continuous function from the topological space  $(X, \tau)$  into the topological space  $(Y, \sigma)$  and let  $V$  be any closed set of  $Y$ . Since  $f$  is  $\delta$ - $e$ -continuous,  $f^{-1}(V)$  is  $\delta$ - $e$ -closed in  $X$ . By Proposition 3.7,  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ . Therefore  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous. ■

**Theorem 4.5** Every super  $\delta g^*$ - $e$ -continuous function is super  $\delta(\delta g)^*$ - $e$ -continuous. but not conversely.

Proof: Let  $f$  be a super  $\delta g^*$ - $e$ -continuous function from the topological space  $(X, \tau)$  into the topological space  $(Y, \sigma)$  and let  $V$  be any closed set of  $Y$ . Since  $f$  is super  $\delta g^*$ - $e$ -continuous,  $f^{-1}(V)$  is super  $\delta g^*$ - $e$ -closed in  $X$ . By Proposition 3.10,  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ . Therefore  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous. ■

**Theorem 4.6** Every super  $\delta(\delta g)^*$ - $e$ -continuous function is super  $g\delta$ - $e$ -continuous (super  $\delta g^\#$ - $e$ -continuous, super  $rg$ - $e$ -continuous, super  $\pi g$ - $e$ -continuous). but not conversely.

Proof: Let  $f$  be a super  $\delta(\delta g)^*$ - $e$ -continuous function from the topological space  $(X, \tau)$  into the topological space  $(Y, \sigma)$  and let  $V$  be any closed set of  $Y$ . Since  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous,  $f^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed in  $X$ . By Proposition 3.13 (Proposition 3.16, Proposition 3.20, Proposition 3.23),  $f^{-1}(V)$  is super  $g\delta$ - $e$ -closed (super  $\delta g^\#$ - $e$ -closed, super  $rg$ - $e$ -closed, super  $\pi g$ - $e$ -closed) in  $X$ . Therefore  $f$  is super  $g\delta$ - $e$ -continuous (super  $\delta g^\#$ - $e$ -continuous, super  $rg$ - $e$ -continuous, super  $\pi g$ - $e$ -continuous). ■

**Theorem 4.7** Let  $f : X \rightarrow Y$  be strongly  $\delta(\delta g)^*$ - $e$ -continuous and  $g : Y \rightarrow Z$  continuous. Then  $g \circ f : X \rightarrow Z$  is

super  $\delta(\delta g)^*$ - $e$ -continuous.

Proof: Let  $V$  be closed in  $Z$ . Since  $g$  is continuous,  $g^{-1}(V)$  is closed in  $Y$ . Since  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous,  $f^{-1}(g^{-1}(V))$  is super  $\delta(\delta g)^*$ - $e$ -closed set in  $X$ . But  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ . Hence  $(g \circ f)^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed set in  $X$ . Therefore  $g \circ f$  is super  $\delta(\delta g)^*$ - $e$ -continuous. ■

**Theorem 4.8** Let  $f : X \rightarrow Y$  be  $\delta$ - $e$ -continuous and let  $g : Y \rightarrow Z$  be continuous. Then  $g \circ f : X \rightarrow Z$  is super  $\delta(\delta g)^*$ - $e$ -continuous.

Proof: Let  $V$  be closed in  $Z$ . Since  $g$  is continuous,  $g^{-1}(V)$  is closed in  $Y$ . Since  $f$  is  $\delta$ - $e$ -continuous,  $f^{-1}(g^{-1}(V))$  is  $\delta$ - $e$ -closed set in  $X$ . By proposition 3.4 (every  $\delta$ - $e$ -closed set is super  $\delta(\delta g)^*$ - $e$ -closed),  $f^{-1}(g^{-1}(V))$  is super  $\delta(\delta g)^*$ - $e$ -closed set. Hence  $(g \circ f)^{-1}(V)$  is super  $\delta(\delta g)^*$ - $e$ -closed set in  $X$ . Therefore  $g \circ f$  is super  $\delta(\delta g)^*$ - $e$ -continuous. ■

**Theorem 4.9** The following statements hold for functions  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$

- (i) If  $f$  is super  $\delta g^*$ - $e$ -continuous and  $g$  is continuous, then  $g \circ f$  is super  $\delta(\delta g)^*$ - $e$ -continuous.
- (ii) If  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous and  $g$  is continuous, then  $g \circ f$  is super  $g\delta$ - $e$ -continuous.
- (iii) If  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous and  $g$  is continuous, then  $g \circ f$  is super  $rg$ - $e$ -continuous.

- (iv) If  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous and  $g$  is continuous, then  $g \circ f$  is Super  $\delta g^\#$ - $e$ -continuous.  
 (v) If  $f$  is super  $\delta(\delta g)^*$ - $e$ -continuous and  $g$  is continuous, then  $g \circ f$  is super  $\pi g$ - $e$ -continuous.

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