A SIMULATION STUDY ON M/M/C QUEUEING MODELS

S.Shanmugasundaram[1] ,sundaramsss@rediffmail.com
P.Banumathi[2] **,** banumathi_p@rediffmail.com.
[1]Department of Mathematics, Government Arts College, Salem – 636 007.India
[2]Department of Mathematics, Government Arts College, Salem – 636 007.India

ABSTRACT:

In this paper we analyses the multiserver queueing system using Monte Carlo simulation in southern railway. The main aim of this paper is future behavior of Southern Railway and how to reduce the queue length and system length, queue time and system time which is compared to analytical method. The numerical examples are also given the feasibility of simulation model.

KEYWORDS:

Multi channel queueing model, Monte-Carlo simulation, probability distribution, queue length, system length and queue time and system time.

INTRODUCTION**:**

The history of queueing theory goes back nearly 100 years. It was born with the work of A.K. Erlang who published in 1909 his paper, the theory of probabilities and telephone conversations[1].

His most important work, solutions of some problems in the theory of probabilities of significance in automatic telephone exchanges was published in 1917, which contained formulas for loss and waiting probabilities which are known as Erlang's loss formula (or Erlang B-formula) and delay formula (or Erlang C- formula) respectively. Erlang's motivation was to develop tools for the analysis and design of telephone systems an application that continues to the present day to motivate research in queueing theory.

The field of telephone traffic was further developed by Molins (1927) and Thornton D-fry (1928) however it was only after World War II that this early work was extended to other general problems involving queues or waiting lines.

Waiting lines or queues are omnipresent. Businesses of all types, industries, schools, hospitals, cafeterias, book stores, libraries, banks, post offices, petrol pumps, theatres – all have queueing problems.

Waiting line problem arise either because

1. There is too much demand on the facilities so that we say that there is an excess of waiting time or inadequate number of service facilities.
2. There is too less demand, in which case there is too much idle facilities time or too many facilities.

Queueing system can be classified into the following characteristics, the input pattern(or arrival),the service mechanism(or pattern),the queue discipline. Many people's is done the research work on applications of queueing theory, Leslie Edie, design the toll booths and derived the traffic delays based on homogeneous vehicles in toll plaza [2]. Belenky, design and optimized the transportation system using by operations research method [3].The analysis methodology of toll plazas originates with the research work by Woo and Hoel [4]. Aycin, Murat develops a methodology for finding the solutions of capacity and delay for the toll plazas using the manual calculation [5]. Klodzinski and Al-Deek using simulation data and determined the variation of delay distribution due to the peak hour [6].In a similar way done the work on M/M/1 queueing models and future behavior of toll plaza [7][8].

Simulation is a way of model random events such that simulated outcomes closely match real world outcomes. By observing simulated outcomes, researches gain insight on the real world. Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of either understanding the behavior of the system and/or evaluating various strategies for the operation of the system. When a situation is affected by random variables it is often difficult to obtain closed from equations that can be used for valuation. Simulation is a very general technique for estimating statistical measures of complex system.

In 1940's Stan Ulam while playing solitaire tries to calculate the like hood of winning based on the initial layout of the cards. After exhaustive combinational calculations, he decided to go for a more practical approach of trying out many different layouts and observing the number of successful games. He realized that computers could be used to solve such problems.Ulam and Von Neumann suggested that aspects of research into nuclear fission of Los Alamos could be aided by use of computer experiments based on change. The project was top secret so Von Neumann chose the name Monte Carlo Simulation[9]. Monte Carlo Simulation is now a much used scientific tool for problems that are analytically intractable and for which experimentation is too time consuming, costly or impractical.

Random numbers are useful for a variety of purposes, such as generating data encryption keys, simulating and modeling complex phenomena and for selecting random samples from larger data sets. They have also been used aesthetically, for example in literature and music, and are of course ever popular for games and gambling. In carryout Monte Carlo simulation, one needs generate random numbers to obtain random observations from a probability distribution.

There are many types[11] of random numbers 1) 'Real' random numbers :uses a physical source of randomness 2) Pseudorandom numbers: deterministic sequence that passes tests of randomness.3)Quasi random numbers: well distributed points (low discrepancy).A random number is a number in sequence of numbers whose probability of occurrence is the same as that of any other numbers in the sequence.

One of the most habitual complaints in ticker counter at railway station is long queue and waiting times at the ticket counter, shorter perceived waiting times decreases service experience, customer loyalty and potentially increase the level of the customer satisfaction.

The perception of waiting time is important, because the number of tickets a customer buys has an influence on the tolerance towards waiting. This is a reason to implement multiserver queuing system to avoid unhappiness to the customer so that the customer need not to be wait in long queue and also don't loose their precious time.

The range of possible expected waiting times depends [10],in a single server queue, the queue length was very high and best case in this study of M/M/C, situation on queuing time. The waiting times are impenetrable to determine exactly. Already we had seen the single server queueing system using Monte Carlo simulation, the study revealed that the queue lengh was very high.

Therefore, M/M/C is selected and using Monte Carlo simulation as follows:-

ARRIVAL DISTRIBUTION

INTER. ARRIVAL	PROB.
1	0.071
1.3	0.100
2	0.141
2.3	0.188
3	0.219
3.3	0.281

SERVICE DISTRIBUTION

INTER SERVICE	PROB.
1	0.071
1.3	0.106
2	0.145
2.3	0.192
3	0.218
3.3	0.268

TAG NUMBER TABLE (ARRIVAL)

INTER ARRIVAL	PROB	CUM PROB.	TAG. NO.
1	0.071	0.071	000-070
1.3	0.100	0.171	071-170
2	0.141	0.312	171-311
2.3	0.188	0.500	312-499
3	0.219	0.719	500-718
3.3	0.281	1.000	719-999

TAG NUMBER TABLE(SERVICE)

INTER SERVICE	PROB.	CUM PROB.	TAG NO.
1	0.071	0.071	000-070
1.3	0.106	0.177	071-176
2	0.145	0.322	177-321
2.3	0.192	0.514	322-513
3	0.218	0.732	514-731
3.3	0.268	1.000	732-999

SIMULATION TABLE

TRIAL NO.	*RANDOM NO. (Arrival time)*	*INTER ARRIVAL TIME(In Hrs)*	*RANDOM NO.(Service time)*	*INTER SERVICE TIME (In hrs.)*	*CLOCK ARRIVAL TIME*	*SERVER - 1*		*SERVER - 2*		*CUSTOMER WAITING TIME IN QUEUE*	*WAITING TIME*		***QUEUE LENGTH***
						BEGIN	END	BEGIN	END		SER-1	SER -2	
1	382	2.3	991	3.3	2.3	2.3	6	-	-	-	2.3	-	-
2	322	2.3	013	1	5	-	-	5	6	-	-	5	-
3	228	2	526	3	7	7	10	-	-	-	1	-	-
4	653	3	603	3	10	-	-	10	13	-	-	4	-
5	009	1	588	3	11	11	14	-	-	-	1	-	-
6	237	2	409	2.3	13	-	-	13	15.3	-	-	-	-
7	112	1.3	461	2.3	14.3	14.3	17	-	-	-	0.3	-	-
8	463	2.3	197	2	17	-	-	17	19		-	1.3	
9	628	3	757	3.3	20	20	23.3	-	-	-	3	-	-
10	572	3	124	1.3	23	-	-	23	0.3	-	-	4	-
11	031	1	826	3.3	24	24	3.3	-	-	-	0.3	-	-
12	318	2.3	042	1	2.3	-	-	2.3	3.3	-	-	2	-
13	565	3	854	3.3	5.3	5.3	9	-	-	-	2	-	-
14	293	2	977	3.3	7.3	-	-	7.3	11	-	-	4	-
15	039	1	521	3	8.3	8.3	11.3	-	-	0.3	-	-	1
16	756	3.3	374	2.3	12	-	-	12	14.3	-	-	1	-
17	847	3.3	485	2.3	15.3	15.3	18	-	-	-	4	-	-
18	076	1.3	451	2.3	17	-	-	17	19.3	-	-	2.3	-
19	572	3	664	3	20	20	23	-	-	-	2	-	-
20	897	3.3	133	1.3	23.3	-	-	23.3	1	-	-	4	-
21	338	2.3	097	1.3	2	2	3.3	-	-	-	3	-	-
22	453	2.3	533	3	4.3	-	-	4.3	7.3	-	-	3.3	-
23	969	3.3	016	1	8	8	9	-	-	-	4.3	-	-
24	139	1.3	086	1.3	9.3	-	-	9.3	11	-	-	2	-

25	376	2.3	818	3.3	12	12	15.3	-	-	-	3	-	-
26	062	1.3	836	3.3	13.3	-	-	13.3	17	-	-	2.3	-
27	879	3.3	154	1.3	17	17	18.3	-	-	-	1.3	-	-
28	042	1	009	1	18	-	-	18	19	-	-	1	-
29	634	3	033	1	21	21	22	-	-	-	2.3	-	-
30	435	2.3	006	1	23.3	-	-	23.3	0.3	-	-	4.3	-
31	547	3	747	3.3	2.3	2.3	6	-	-	-	4.3	-	-
32	636	3	203	2	5.3	-	-	5.3	7.3	-	-	5	-
33	513	3	036	1	8.3	8.3	9.3	-	-	-	2.3	-	-
34	021	1	319	2	9.3	-	-	9.3	11.3	-	-	2	-
35	242	2	147	1.3	11.3	11.3	13	-	-	-	2	-	-
36	029	1	926	3.3	12.3	-	-	12.3	16	-	-	1	-
37	727	3.3	075	1.3	16	16	17.3	-	-	-	3	-	-
38	687	3	284	2	19	-	-	19	21	-	-	3	-
39	803	3.3	346	2.3	22.3	22.3	1	-	-	-	5	-	-
40	028	1	044	1	23.3	-	-	23.3	0.3	-	-	2.3	-
41	321	2.3	774	3.3	2	2	5.3	-	-	-	1	-	-
42	046	1	716	3	3	-	-	3	6	-	-	2.3	-
43	494	2.3	348	2.3	5.3	5.3	8	-	-	-	-	-	-
44	216	2	427	2.3	7.3	-	-	7.3	9	-	-	1.3	-
45	902	3.3	086	1.3	11	11	12.3	-	-	-	3	-	-
46	699	3	958	3.3	14	-	-	14	17.3	-	-	5	-
47	317	2.3	438	2.3	16.3	16.3	19	-	-	-	4	-	-
48	288	2	583	3	18.3	-	-	18.3	21.3	-	-	1	-
49	859	3.3	577	3	22	22	1	-	-	-	3	-	-
50	658	3	806	3.3	1	-	-	1	4.3	-	-	3.3	-
TOTAL		121		119.3						0.3	59	68.3	1

From simulation table

1.Average service time =1.18

2.Average arrival time = 1.366

3.Average queue length = 0.02

4.Average waiting time of a customer = 0.006

5.Time a customer spends in the system = 2.386+0.006

=2.392

6.Average waiting time of Server -1 =1.18

Server -2 =1.366

Analytical method

1. Excepted number of customer in the system

   $L_S = 1.33$

2. Excepted number of customers waiting in the queue

   $L_q = 0.33$

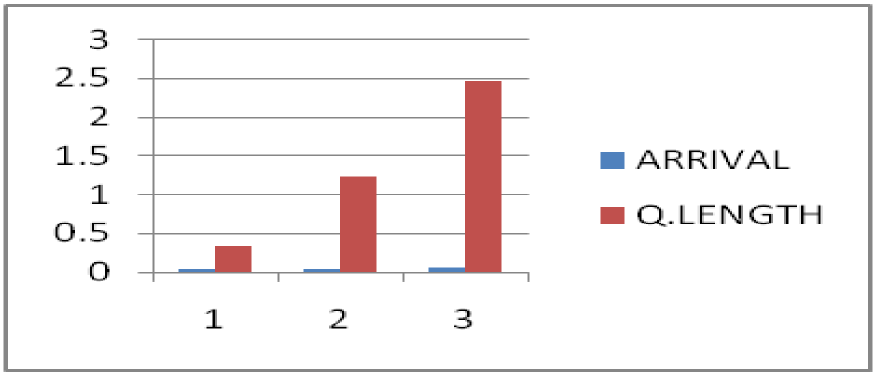
3. Average time a customer spends in the system

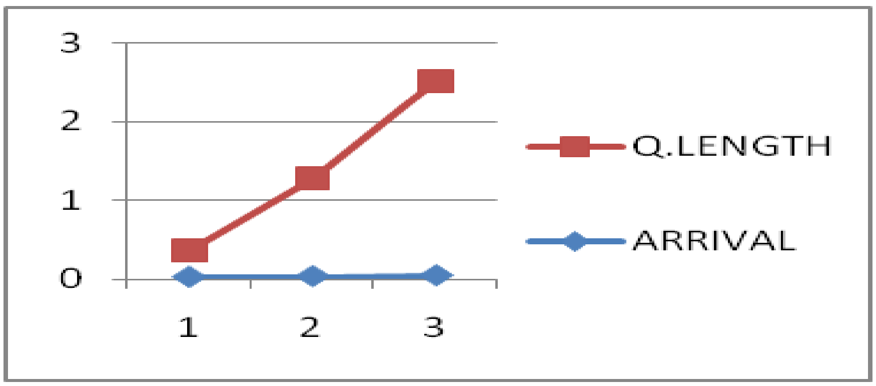
   $W_s = 1.33$ hrs

4. Average waiting time of a customer in the queue

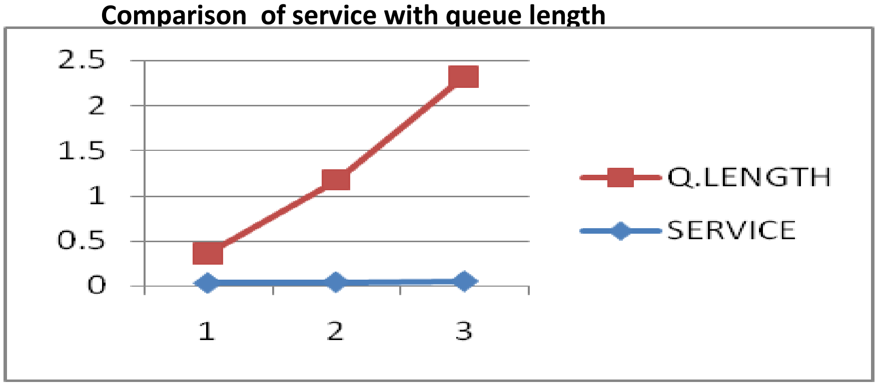
   $W_q = 0.18$ hrs

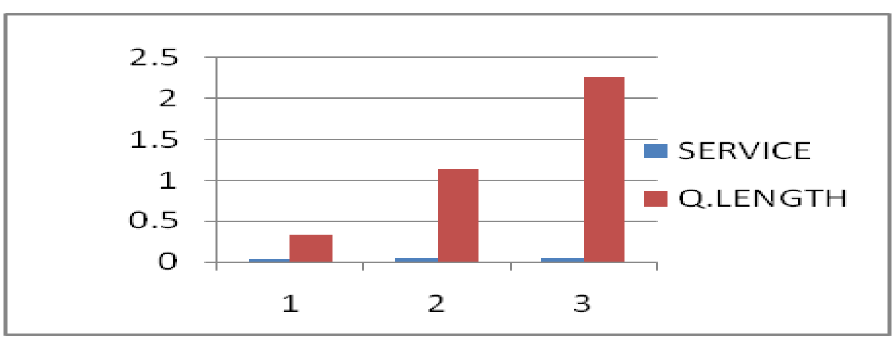
Comparison of Arrival with queue length





Comparison of service with queue length





CONCLUSION

The study about queues and waiting times at railway ticket counter are important, so this study is revealed that intuitively understandable that the analytical method. A better solution to reduce perceived waiting times could be, average waiting time of the customer, average queue length, time a customer spent in the system. The possibly decreases the passive waiting time in the ticket counter.

Therefore Multiserver queueing system is better to compare the single server queueing system and also simulation is best method.

References :

1. Erlang A.K. The theory of probabilities of telephone conversation, Nyt Jindsskriff mathematics, B 20,33-39(1909).
2. Edie, A. C. (1954). Traffic delays at toll booths, Journal of Operations Research Society of America, 2, 107-138.
3. Belenky, A.S. (1998). Operations Research in Transportation Systems – Ideas and Schemes of Optimization Methods, lower Academic Publisher.
4. Woo T, and Lester Hoel. “Toll Plaza Capacity and Level of Service.” Transportation Research Record. 1320. (1991):119-127.Print.
5. Aycin, Murat. “Simple Methodology for Evaluating Toll Plaza Operation.” Transportation Research Record. 1988. (2006): 92-101.Print.
6. Klodzinski, Jack, and Haitham Al-Deek. “Transferability of a Stochastic Toll Plaza Computer Model.” Transportation Research Record. 1811.(2002):40-49. Print.
7. S.Shanmugasundaram and S.Punitha. 2014. “A Simulation Study on toll gate system in M/M/1 Queueing Models”. IOSR-JM PP 01-09.
8. S.Shanmugasundaram and S.Punitha. “Future behavior of toll plaza using Monte Carlo simulation” GJPAM-ISSN 0973-1768 volume 11.

9. A brief history- Monte Carlo simulation-Lancaster university, Mathematics and statistics-STOR-i.
10. S.Shanmugasundaram & P Banumathi. “A simulation study on railway system in M/M/1 queueing models” GJPAM-ISSN 0973-1768 volume 11.
11. Prof.Michael Mascagni-A practitioner’s overview ‘Random numbers Generation’.