# FUZZY CLOSED SUBSETS IN FUZZY TOPOLOGICAL SPACES

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**Abstract**

The boundary of a fuzzy subset was investigated by R.H. Warren in 1977. N. Levine introduced and studied generalized closed sets in general topology in the year 1970. These two concepts motivated to investigate boundary closed fuzzy sets in fuzzy topological spaces. The present paper deals with boundary closed fuzzy sets and related ideas. The concepts of boundary continuous maps, boundary open maps, boundary closed maps and related concept have been introduced and studied.

**Key words:** boundary continuous maps, boundary open maps, boundary closed maps.

## Introduction

The boundary of a fuzzy subset was introduced and studied by R.H. Warren in 1977. This concept was further investigated by Wu Guang- qing and Zheng Chong- you in 1991.

N. Levine introduced the concept of generalized closed sets in general topology in the year 1970 and it was further investigated and many useful results in general topology were obtained by many researchers.

This idea of N. Levine motivated the generalization of the concept of closed fuzzy sets in fuzzy topological spaces into the concept of boundary closed fuzzy sets, using the concept of the boundary of fuzzy subset by R.H. Warren.

The present paper deals with this concept and some related concepts. Several results have been obtained.

## 1. Definitions

The following concept of boundary of a fuzzy set was introduced by R.H. Warren [6]

**1.1 Definition** [6]. Let A be a fuzzy set in afts (X, T). The fuzzy boundary of A is defined as the infimum of all closed fuzzy sets d in X with the property:

$d(x)\geq\overline{A}(x)$ for all $x\in X$ for which $(\overline{A}\wedge(1-\overline{A}))(x)>0$.

bd (A) = $A^b$ = inf {d:d is closed fuzzy set and $d(x)\geq\overline{A}(x)$ for all x $\in$X for which $(\overline{A}\wedge(1-\overline{A}))(x)>0$ }.

Note that bd (A) is a closed fuzzy set and $bd(A)\leq\overline{A}$. The following concept is due to P. Sundaram [5].

**1.2 Definition** [5]. Let X be a fts. A fuzzy set A in X is said to be generalized clossed (g-closed) fuzzy set in X if cl (A) ≤ U and U is an open fuzzy set in X.

K.K. Azad [1] defined regular open and regular closed fuzzy sets in fts as below.

**1.3 Definition** [1]. A fuzzy subset A of afts X is said to be

(i) a regular open fuzzy set in X if int (cl (A)) = A and

(ii) a regular closed fuzzy set in X if cl (int A) = A.

C.L. Chnag [4] defined compactness in fts as below.

**1.4 Definition** [4]. A fts (X, T) is compact iff each open cover of X has a finite sub cover. The following definition is due to G. Balasubramanian and P. Sundaram [2].

**1.5 Definition** [2]. Afts X is said to be fuzzy $T_{1/2}$-space if every g-closed fuzzy set is a closed fuzzy set in X.

**2. Boundary Closed Fuzzy Sets in Fts**

**2.1 Definition:** A fuzzy set A of a fuzzy topological spaces X is called boundary closed (b-closed) fuzzy set if bd 9A) ≤ G whenever A ≤G and G is an open fuzzy set.

For reasons of space, the proofs of the results given in this paper have been omitted.

**2.2 Theorem.** Every closed fuzzy set is a b-closed fuzzy set in any fts X.

The converse of the above theorem need not be true as seen from the following example.

**2.3 Example.** Let X = [0, 1] and A be a fuzzy subset of X defined by

$$A(x) = \begin{cases} 0.6 & if\ x = 2/3 \\ 0 & otherwise \end{cases}$$

Consider *t* = {0, 1, A}. Then (X, *t*) is afts. Let B be a fuzzy subset of X defined by

$$B(x) = \begin{cases} 0.6 & if\ x = 2/3 \\ 0 & otherwise \end{cases}$$

Then B is a b-closed fuzzy set.

For B ≤1 where 1 is an open fuzzy set., we have bd (B) ≤1. Further B is not a closed fuzzy set. Hence B is a bn-closed fuzzy set, which is not a closed fuzzy set.

**2.4 Theorem.** Every g-closed fuzzy set is b-closed fuzzy set.

The converse of the above theorem is true if cl(A) ∧cl(1-A) > 0 for any fuzzy set A.

**2.5 Theorem:** If A is a b-closed fuzzy set in a fts X and cl(A) ∧ cl(1-A) > 0, then A is a g-closed fuzzy set.

**2.6 Theorem.** If a fuzzy set A of a fuzzy topological space X is both open fuzzy set and b-closed fuzzy set, then it is a closed fuzzy set.

**2.7 Corollary:** If a fuzzy set A in afts X is both open fuzzy set and b-closed fuzzy set then it is g-closed fuzzy set.

**2.8 Corollary:** If A is both open fuzzy set and b-closed fuzzy set in afts X then it is regular open fuzzy set and regular closed fuzzy set in X.

**2.9 Theorem:** Let X be a fts and A be a fuzzy subset of X such that bd(A) ∧ (1-bdA) = 0. Then A is b-closed fuzzy set iffbd(A)∧ (1-A) contains no non zero closed fuzzy set.

**2.10 Theorem:** The union of any two b-closed fuzzy sets of afts X is b-closed fuzzy set.

**2.11 Remarks.**

(1) Finite union of b-closed fuzzy sets in a b-closed fuzzy set.

(2) Intersection of b-closed fuzzy sets need not be b-closed.

**2.12 Example.** Let X = {a, b, c}. Fuzzy sets A, B and C in X are defined as follows: A ={(a, 0.3), (b, 0.5), (c, 0.6)}, B= {(a, 0.5), (b, 0.4), (c, 0.6)} and C = {(a, 0.2), (b, 0.5), (c, 0.7)}. Consider the fts (X, T) where T = {0, 1, A}. The fuzzy sets B and C are b-closed. Now B ≤1 implies bd (B) = 1≤1 and C≤1 implies bd(C) = 1≤1. Also, D = B∧C = {(a, 0.2), (b, 0.4), (c, 0.6). D≤A, 1, which are open in X. Cl(D) = 1, cl(1-D) = 1; therefore cl(D) ∧cl(1-D) > 0. Thus bd(D) = cl(D)-1 >A. Therefore D = B∧C is not a b-closed fuzzy set. Hence the intersection of any two b-closed fuzzy sets need not be a b-closed fuzzy set.

S.S. Benchalli and Jenifer Rodrigues [3] proved that a closed crisp subspace of a compact fts is compact. Therefore it follows from 2.6 that an open b-closed crisp subspace of a compact fts is also compact.

**2.13 Definition.** A fuzzy set A of afts X is called b-open fuzzy set if its complement (1-A) is b-closed fuzzy set.

**2.14 Theorem.** Let A be any fuzzy set such that $\overline{A} \wedge (\overline{1-A}) > 0$, then A is b-open iff F≤ 1-bd(A) whenever F ≤A and F is closed fuzzy set.

**2.15 Remark:** The above proof also follows immediately in view of the result bd(A) = 0 = bd (1-A) iff $\overline{A} \wedge (\overline{1-A}) > 0$ [6, Theorem 5.2 (V)].

**2.16 Theorem:** Every open fuzzy set is b-open fuzzy set.

The converse of the above theorem need not be true as.

**2.17 Example:** Let X = [0, 1] and A be a fuzzy subset of X defined by

$$A(x) = \begin{cases} 0.5 & if\ x = 2/3 \\ 0 & otherwise \end{cases}$$

Consider T = {0, 1, A}. Then (X, T) is a fuzzy topological space. Let B be a fuzzy subset of X defined by

$$B(x) = \begin{cases} 0.6 & if\ x = 2/3 \\ 0 & otherwise \end{cases}$$

and 1-B ≤ 1 where 1 is an open fuzzy set. Then we have bd(1-B) ≤ 1. And so l-B is b-closed fuzzy set. Thus B is b-open fuzzy set. Further B is not an open fuzzy set. Hence B is a b-open fuzzy set which is not an open fuzzy set.

**2.18 Remark.** Every g-open fuzzy set is b-open (follows from 2.4).

**2.19 Theorem.** The intersection of any two b-open fuzzy sets is b-open.

**2.20 Remark.** It can be verified from example 2.12, that union of two b-open fuzzy sets need not be b-open.

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