Analysis of Flow of an Incompressible MHD Third Grade Fluid In An Inclined Rotating Cylindrical Pipes With Isothermal Wall and Joule Heating

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Abstract

In this paper, we considered the analysis of flow of an incompressible MHD third grade fluid in an inclined rotating cylindrical pipes with isothermal wall and Joule heating. The governing equations of the flow field are solved using perturbation method. Results show that the Eckert and the Grashhof parameters reduce the velocity of the fluid flow, while the magnetic field parameter, the Grashhof number, the Eckert number and the third grade parameter increase the temperature of the system. . It is further discovered that the temperature profiles converges to zero at the point where $R = 2$ *,which shows that isothermal nature of the outer cylinder.*

Keywords: *Incompressible MHD, isothermal wall, Joule heating.*

1. Introduction

In the study of the flow of an incompressible magnetohydrodynamic third grade fluid in a rotating cylindrical annulus with isothermal wall and Joule heating, many investigators have considered the flow of third grade fluid in different forms. Some of the foremost works are Fosdick and Rajagopal (1980), who examined the thermodynamics and stability of fluids of third grade and showed restrictions on the stress constitutive equation was concern with the relation between thermodynamics and stability for a class of non-Newtonian incompressible fluids of the differential type, and further introduced the additional thermodynamical restriction that the Helmholtz free energy density be at a minimum value when the fluid is locally at rest. They gave detailed attention to the special case of fluids of grade 3 and arrived at fundamental inequalities which restricts its temperature dependent. They discovered that these inequalities requires that a body of such a fluid be stable in the sense that its total kinetic energy must tend to zero in time, no matter what its previous mechanical and thermal fields, provided it is both mechanically isolated and immersed in a thermally passive environment at constant temperature from some finite time onward.

Rajagopal *et al* (1986) examined the flow past an infinite porous flat plates with suction. They discovered that the non-Newtonian fluid mechanics affords an excellent opportunity for studying many of the mathematical methods which have been developed to analyse non-linear problems in mechanics. They established an existence theorem using the shooting and investigated the problem using perturbation analysis and numerical method and concluded that since perturbation method did not converge suitably, it was not the appropriate tool for the problem. Obi *et al* (2017) examined annular flow of an incompressible MHD third grade fluid in a rotating concentric cylinders with isothermal walls and Joule heating.

Okoya (2011) considered the disappearance of criticality for reactive third grade fluid with model viscosity in a flat channel. Mahmood and Nargis (2012) on thin film flow of a third grade fluid through porous medium over an inclined plane, employed the perturbation and homotopy perturbation methods and the results obtained from the two methods are in close agreement. Masoudi and Christie (1995) analysed the effect of temperature-dependent viscosity for the three separate cases treated by Szeri and Rajagopal (1998).

Aiyesimi *et al* (2014) dealt with the effects of magnetic field on the MHD flow of third grade fluid through inclined channel with Ohmic heating. They analysed the Couette flow, Poiseuille and Couette-Poiseuille flow and solved the resulting non-linear differential equation by employing regular perturbation technique.

Makinde (2005) examined the hydrodynamic ally and thermally developed Reynolds viscosity liquid film along an inclined heated plate: an exploitation of Hermite-pade approximation technique. Ayub, et al (2003) examined the exact flow of third grade fluid past a porous plate using homotopy analysis method. Yurusoy and Pakdemirli (2002) considered approximate analytical solutions for the flow of a third grade fluid in a pipe. Makinde (2007) studied thermal stability of a reactive third grade fluid in a cylindrical pipe: An exploitation of Hermite Pade approximation technique.

Baldoni *et al* (1993) studied the helical flow of a third grade fluid between eccentric cylinders, using a domain perturbation approach. They discovered a secondary flow contrary to the initial analysis. The consistency of the slow flow approximation has been tested considering the behaviour of the fluid at intermediate and high Reynolds number.

According to Rajagopal and Mollica (1999) on secondary deformation due to axial shearing of annular region between two eccentrically placed cylinders followed the works of Fosdick and kao (1978) and extended a conjecture of Ericksen's (1956) for non linear fluid, to nonlinear elastic solids and showed that unless the material modulli of an isotropic elastic material satisfied certain special relations, axial shearing of cylinders would be necessarily accompanied by secondary deformation if the cross-section were not a circle or the annular region between concentric circles.

2. Mathematical Formulation:

Following Feiz-Dizaji et al (2007), we consider the steady flow of an incompressible MHD third grade fluid in a rotating cylindrical annulus with isothermal wall. Pressure-gradient was assumed to have induced the fluid motion. For magnetohydrodynamically developed flow, both the velocity and the temperature fields depend on r only. Considering [5, 10, 14] and neglecting the reacting viscous fluid assumption, the governing equations for the momentum and energy balance can be represented as:

$$\mu\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}\right)+\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(6\frac{d^2u}{dr^2}-\frac{2}{r}\frac{du}{dr}+2\frac{u}{r^2}\right)+\rho g\beta\left(T-T_0\right)\sin\phi-\sigma B_0^2u=0 \quad (1)$$

$$k\left(\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}-\frac{T}{r^2}\right)+\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(\delta+\beta_3\left(\frac{du}{dr}-\frac{u}{r}\right)^2\right)+\sigma B_0^2u^2=0 \quad (2)$$

subject to the conditions

$$\frac{dT}{dr}\left(r_1\right)=0,\ T\left(r_2\right)=T_0\ \ u=r_1\omega_1\ \ at\ \ r=r_1;\ u=r_2\omega_2\ \ at\ \ r=r_2 \quad (3)$$

We introduce the following non-dimensional variables into equations for non-dimensionalisation

$$\theta=\frac{T}{T_0},\bar{r}=\frac{r}{a},\bar{u}=\frac{u}{u_0}\ \ \sigma=\frac{\beta_3u^2}{a^2\delta},\ r=r_1\bar{r}\ and\ u=r_1\omega\bar{u} \quad (4)$$

where

$r_1<r_2$ and r_1 is the radius of the inner cylinder, r_2 is the radius of the outer cylinder, ω_1 is the angular velocity of the inner cylinder , ω_2 is the angular velocity of the outer cylinder T is the temperature of the cylinder, T_0 is the plate temperature, μ is the coefficient of dynamic viscosity of the fluid, u is the fluid velocity, u_0 is the characteristic fluid velocity, B_0 is the magnetic field effect, β_3 is the material coefficient and σ is the heat reaction.

Then substituting equation (4) into equations $(1)\ to\,(3)$, we obtain

$$\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}\right)+\mu\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(6\frac{d^2u}{dr^2}-\frac{2}{\bar{r}}\frac{du}{dr}+\frac{2u}{r^2}\right)+\delta\theta-\tau u=0 \quad (5)$$

$$\left(\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-\frac{\theta}{r^2}\right)+\left(\frac{du}{dr}-\frac{u}{r}\right)^2+B_r\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^4+B_r\tau u^2=0 \quad (6)$$

3. METHOD OF SOLUTION

In order to solve equation (5), we introduce perturbation series as follows:

$$u = u_0 + \beta u_1 + \beta u_2 + 0(\beta)^3, \theta = \theta_0 + \beta\theta_1 + \beta\theta_2 + 0(\beta)^3 \text{ and } \tau = \beta M \qquad (7)$$

Substituting equation (7) into equation (5), we obtain order β^0, β and β^2 problems as follows:

$$\beta^0 : \frac{d^2u_0}{dr^2} + \frac{1}{r}\frac{du_0}{dr} - \frac{u_0}{r^2} = 0 \qquad (8)$$

$$\beta : \frac{d^2u_1}{dr^2} + \frac{1}{r}\frac{du_1}{dr} - \frac{u_1}{r^2} + \gamma\frac{du_0}{dr}\frac{d^2u_0}{dr^2} + G\theta_0 - Mu_0 = 0 \qquad (9)$$

$$\beta^2 : \left(\left(\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \frac{u_2}{r}\right) - \frac{12u_0}{r}\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{12u_0}{r}\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{12u_0}{r}\left(\frac{du_0}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right)\right.$$

$$+\frac{6u_0^2}{r^2}\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right) - \frac{12u_0u_1}{r^3}\left(\frac{du_0}{dr}\right) + \frac{6u_1}{r^2}\left(\frac{du_0}{dr}\right)^2 + \frac{12u_0}{r^2}\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right) - \frac{6}{r}\left(\frac{du_0}{dr}\right)^2\left(\frac{du_1}{dr}\right)$$

$$+6\left(\frac{du_0}{dr}\right)^2\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right) + \frac{6u_0^2u_1}{r^4} + \frac{12u_0u_1}{r^2}\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{6u_0^2}{r^3}\left(\frac{du_1}{dr}\right) + 12\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right)$$

$$\left.-\frac{12u_1}{r}\left(\frac{du_0}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right)\right) + G\theta_1 - Mu_1 = 0 \qquad (10)$$

Solving equations (8), (9) and (10), using the condition (3), we obtain

$$u_0(r) = \frac{R(R-\omega)}{(-1+R^2)r} + \frac{(\omega R - 1)r}{-1+R^2} \qquad (11)$$

that is

$$u_0(r) = \frac{A}{r} + Br \qquad (12)$$

where

$$A = \frac{R(R-\omega)}{(-1+R^2)} \text{ and } B = \frac{(\omega R-1)}{-1+R^2} \qquad (13)$$

$$u_1(r) = r\left(-\frac{721}{4374} - \frac{2}{3}GA\ln(2) - \frac{127}{1080}M - \frac{5}{8}GB - \frac{10}{27}M\ln(2) - \frac{80}{81}\ln(2) + \frac{1}{4}GA\right)$$

$$+\frac{1}{r}\left(\frac{2611}{1458} + \frac{2}{3}GA\ln(2) + \frac{49}{270}M + \frac{1}{2}GB + \frac{10}{27}M\ln(2) + \frac{80}{81}\ln(2)\right) + \frac{1}{87480}\frac{1}{r^5}\left(-72000r^3\right.$$

$$+24300M\ln(r)r^6 + 43740GA(r)r^6 - 21870GAr^6 + 8505Mr^8 - 40000 - 12150Mr^6$$

$$\left.-1944Mr^6 + 10935GBr^8 + 2160r^9 + 64800\ln(r)r^6 - 32400r^6\right) \tag{14}$$

$$u_2(r) = \frac{1}{661348800}\frac{1}{r^9}\left(653184000M\ln(2)\ln(r)r^{10} + 440899200GB\ln(r)r^{10} - 3124396800GA\ln(2)r^{10}\right.$$

$$+122472000M^2\ln(2)\ln(r)r^{10} - 1306368000GA\ln(2)r^7 - 146994000GAr^9 + 41334300MGAr^{10}$$

$$+103333575MGBr^{10} - 3509184000r^7 - 10124587200r^8 + 2880000000r^3 + 165337200MGB\ln(r)r^{10}$$

$$+587865600GA\ln(2)\ln(r)r^{10} + 41334300MGAr^{12}\ln(r) - 55112400MGA\ln(2)r^{12}$$

$$-2069776800GAr^8 - 110224800G\theta_1 r^8 + 330674400r^{10}G\theta_1 - 220449600G\theta_1 r^{11}$$

$$+30618000M^2\ln(2)r^8 + 3147984000M\ln(2)r^8 - 29243220000r^4 + 612360000GAr^6$$

$$+30240000000r^6 + 30248800000Mr^6 + 3717025200GBr^8 + 1034208000Mr^9 + 341105580Mr^8$$

$$+3628800000r^9 - 21432600r^{14}M + 6345216Mr^{15} - 571536000Mr^{11} - 53430300Mr^{12}$$

$$-38425590M^2r^{12} + 28796229M^2r^{10} + 2679075r^{14}M^2 - 385685496Mr^{10} - 419904r^{15}M^2$$

$$-296352000r^4M - 816480000GBr^4 - 1935360000\ln(2)r^7 + 1579132800\ln(r)r^{10}$$

$$-4628736000\ln(2)r^{10} - 81648000M\ln(2)r^{12} + 22963500M^2r^{12}\ln(r) - 725760000M\ln(2)r^7$$

$$+37889775MGBr^8 - 10333575MGAr^8 + 5519404800GA\ln(2)r^8 + 1344000000$$

$$-1735776000M\ln(2)r^{10} + 870912000\ln(2)\ln(r)r^{10} + 60011280M^2\ln(r)r^{10}$$

$$+752204880M\ln(r)r^{10} - 30618000M^2\ln(2)r^{12} + 612360000Mr^{12}\ln(r) + 7370190M^2r^8$$

$$+8176896000\ln(2)r^8+3444525MGBr^{14}-51667875MGBr^{12}-31000725MGAr^{12}$$

$$-6531840r^{15}+5959983040r^{10}+97977600GAr^{10}+220449600MGA\ln(2)\ln(r)r^{10}$$

$$-1158381000GBr^{10}-27556200GBr^{14}-110224800GAr^{12}-734832000GBr^{11}$$

$$-1088640000r^4GA\ln(2)-979776000GBr^7-604800000r^4M\ln(2)-355622400Mr^7$$

$$-1612800000r^4\ln(2)+55112400MGA\ln(2)r^8-272160000r^{12}\Big) \qquad (15)$$

4.0 Heat Transfer Analysis

Usng the series (8) in (6) and simplifying, we obtain

$$\beta^0:\frac{d^2\theta_0}{dr^2}+\frac{1}{r}\frac{d\theta_0}{dr}-\frac{\theta_0}{r^2}=0 \qquad (16)$$

$$\beta:\frac{d^2\theta_1}{dr^2}+\frac{1}{r}\frac{d\theta_1}{dr}-\frac{\theta_1}{r^2}+6B_r\frac{u_0^2}{r^2}\left(\frac{du_0}{dr}\right)^2-4B_r\frac{u_0^3}{r^3}\left(\frac{du_0}{dr}\right)+B_r\left(\frac{du_0}{dr}\right)^4-6B_r\frac{u_0}{r}\left(\frac{du_0}{dr}\right)^3$$

$$+\left(\frac{du_0}{dr}\right)^2-\frac{u_0}{r}\left(\frac{du_0}{dr}\right)+\frac{u_0^2}{r^2}+B_r\frac{u_0^4}{r^4}+Mu_0^2=0 \qquad (17)$$

$$\beta^2:\frac{d^2\theta_2}{dr^2}+\frac{1}{r}\frac{d\theta_2}{dr}-\frac{\theta_2}{r^2}-\frac{12B_ru_0}{r}\left(\frac{du_0}{dr}\right)^2\left(\frac{du_0}{dr}\right)+2E_c\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)-\frac{4B_ru_1}{r}\left(\frac{du_0}{dr}\right)^3$$

$$+\frac{4B_ru_0^3u_1}{r^4}+\frac{12B_ru_0u_1}{r^2}\left(\frac{du_0}{dr}\right)^2-\frac{2E_cu_1}{r}\left(\frac{du_0}{dr}\right)-\frac{12B_ru_0^2}{r^3}\left(\frac{du_0}{dr}\right)+\frac{12B_ru_0^2}{r^2}\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)$$

$$-\frac{12B_ru_0^3}{r^3}\left(\frac{du_1}{dr}\right)-\frac{2B_cu_0}{r}\left(\frac{du_1}{dr}\right)+4B_r\left(\frac{du_0}{dr}\right)^3\left(\frac{du_1}{dr}\right)+\frac{2E_cu_0u_1}{r^2}+2B_rMu_0u_1=0 \qquad (18)$$

$$\frac{d\theta}{dr}(1)=0,\ \theta(R)=1 \qquad (19)$$

Solving equations (16), (17) and (18), using the condition (19) we obtain

$$\theta_0(r)=-\frac{R}{(-1+R^2)r}+\frac{Rr}{-1+R^2} \qquad (20)$$

that is

$$\theta_0(r)=\frac{E}{r}+Fr \tag{21}$$

where

$$E=\frac{R}{\left(-1+R^2\right)}\ and\ F=\frac{Rr}{1+R^2} \tag{22}$$

$$\theta_1(r)=r\left(\frac{8}{27}E_c\ln(2)-\frac{962867}{2296350}B_r+\frac{15056}{42525}B_rM-\frac{367}{2025}E_c\right)+\frac{1}{r}+\frac{8}{27}E_c\ln(2)+\frac{1065523}{2296350}B_r$$

$$-\frac{6614}{42525}B_rM+\frac{2594}{6075}E_c-\frac{1}{918540}\frac{1}{r^6}\left(-756000B_rr^6-283500B_rMr^6+2916B_rMr^{12}+324B_rr^{12}\right.$$

$$-19845B_rMr^{11}+6804E_cr^{10}+37044r^{10}B_rM-42525B_rMr^9+18900B_rr^9+264600B_rMr^8$$

$$\left.+378000E_cr^4+340200E_cr^7\ln(r)170100E_cr^7+40000B_r+210000B_rr^3\right) \tag{23}$$

$$\theta_2(r)=-\frac{1}{589161780800}\frac{1}{r^{10}}\left(-359251200000B_rr^{12}+2664766781600B_rr^9+19399564880000E_cr^{10}\right.$$

$$+29785909923B_rM^2r^9-61585920B_rMr^{18}+1563341472000E_cr^8-3126682944000B_rr^{10}$$

$$-84873096000E_cMr^{12}-3182741100E_cr^{15}M-8911675080B_rM^2r^{13}-81356924880B_rM^2r^{12}$$

$$-1220050755B_rM^2r^{17}-20482707168B_rM^2r^{14}+2546192880B_rM^2r^{16}+5486439420B_rM^2r^{15}$$

$$+117879300B_rMr^{17}+1392322932E_cMr^{11}+192836581740B_rM^2r^{11}+138568320B_rM^2r^{18}$$

$$+748268928E_cr^{16}M-2728063800E_cMr^{13}+15900607800B_rMr^{15}-83977213320B_rMr^{13}$$

$$-68163923520B_rMr^{14}+1993787136000B_r\ln(2)r^{11}+21555072000B_rr^{13}\ln(2)$$

$$+237105792000E_c\ln(2)r^{11}+703503662400E_c\ln(r)r^{11}-1724405760000B_r\ln(2)r^{10}$$

$$+862202880000E_c\ln(2)r^{8}+182476800000B_r\ln(2)r^{4}+718502400000B_r\ln(2)r^{7}$$

$$+132024816000B_rMr^{7}-351920230200E_cGAr^{9}-619270482600E_cGBr^{9}-187229750400B_rM\ln(2)r^{9}$$
$$+183007613250B_rMGAr^{11}-872980416000B_rGBr^{10}+363741840000B_rGBr^{7}-269438400000B_rGBr^{8}$$

$$+556794453600B_rGBr^{9}+92378880000B_rGBr^{4}+654735312000E_cGAr^{15}+436490208000E_cGBr^{8}$$

$$-107775360000B_rGAr^{6}+1345806316800B_rGA\ln(2)r^{11}+14549673600B_rr^{13}GA\ln(2)$$

$$+97401981600B_rM^2\ln(2)r^{9}+476007840000B_rM\ln(r)r^{9}-412240752000E_cM\ln(2)r^{9}$$

$$-742033353600E_cGA\ln(2)r^{9}+727483680000B_rGA\ln(r)r^{9}+39083536800r^{13}B_r$$

$$-831409920E_cr^{16}-102643200B_rr^{18}-1235636376480E_cr^{11}+374500756800B_rr^{11}$$

$$-48498912000E_cr^{13}-5388768000B_rr^{15}-24552574200B_rMGBr^{13}+10230239250B_rMGBr^{15}$$

$$+3273676560B_rMr^{16}GB+16368382800B_rMGBr^{12}-32736765600B_rMGBr^{14}+232021826190B_rMGBr$$

$$-660124080000B_rM\ln(r)r^{11}+387991296000E_c\ln(2)\ln(r)r^{11}+71293400640E_cM\ln(r)r^{11}$$

$$+4526561200B_rr^{14}M\ln(r)+86220288000B_rr^{12}M\ln(2)-60354201600B_rr^{14}M\ln(2)$$

$$-40415760000B_rr^{13}M\ln(2)-18187092000B_rM^2\ln(2)r^{13}+32332608000B_rM^2\ln(2)r^{12}$$

$$+16974619200B_rM^2r^{14}\ln(r)-22632825600B_rM^2r^{14}\ln(2)-12124728000B_rMr^{15}\ln(r)$$

$$+3410079750B_rMGAr^{15}-145496736000B_rMGAr^{12}-1023023925B_rMr^{17}GB-16295634432B_rMr^{14}GA$$
$$-363741840000B_rGB\ln(r)r^{11}+196420593600E_cGB\ln(r)r^{11}+145496736000E_cM\ln(2)\ln(r)r^{11}$$

$$+160046409600E_cGA\ln(2)r^{11}-1163973888000B_rGA\ln(2)r^{10}+484989120000B_rBA\ln(2)r^{7}$$

$$+123171840000B_rGA\ln(2)r^{4}+581986944000E_cGA\ln(2)r^{8}+123785894925B_rMGBr^{9}$$

$$-24625322568B_rMGAr^9-804543062400B_rGA\ln(2)r^9+109104053080B_rMr^9+306421315200B_rMr^{12}$$

$$+1302784560000B_rr^7-798336000000B_rr^6-332640000000B_rr^3-538876800000E_cr^7$$

$$-102643200000E_cr^4-349272000000B_rMr^8+491638724280B_rMr^{11}-1489365662400B_rMr^{10}$$

$$-79833600000B_rMr^6+68428800000B_rM\ln(2)r^4-404309159100B_rGBr^9-1576811253600E_cr^9$$

$$-89600000000B_r+33530112000B_rMr^4-32736765600B_rMGA\ln(2)r^{13}$$

$$+58198694400B_rMGA\ln(2)r^{12}+30554314560B_rMr^{14}GA\ln(r)-40739086080B_rMr^{14}GA\ln(2)$$

$$-8184191400B_rMGAr^{15}\ln(r)+10912255200B_rMr^{15}GA\ln(2)+109122552000B_rMGAr^{12}\ln(r)$$

$$+1094699350800B_rGBMr^{11}-301703648400B_rGAr^{11}+10912255200B_rr^{13}GB$$

$$-278262507600E_cGAr^{11}+295994922300E_cGBr^{11}-145496736000B_rr^{14}GAr^{12}$$

$$-109122552000E_cGBr^{12}-454677300B_rr^{17}GB-14549673600B_rr^{14}GB-24552574200E_cGAr^{13}$$

$$-4092095700E_cGBr^{15}-1818709200B_rr^{15}GA+363741840000E_cMr^{10}-118822334400B_rM^2r^{10}$$

$$+158429779200E_cMr^8+26943840000B_rM\ln(2)r^2+323326080000E_cM\ln(2)$$

$$-242494560000B_rM^2\ln(2)r^{10}-1293304320000B_rM\ln(2)r^{10}$$

$$-4546773000B_rM^2r^{15}\ln(r)+161663040000B_rMr^{12}\ln(r)+6062364000B_rM^2r^{15}\ln(2)$$

$$+60623640000B_rM^2r^{12}\ln(r)+161663040000B_rM\ln(2)r^{15}+88914672000E_cM\ln(2)r^{11}$$

$$+1141050240000B_rM\ln(2)r^{11}+147517524000B_rM^2\ln(2)r^{11}-327367656000B_rMGBr^{10}$$

$$-436490208000B_rMGA\ln(2)r^{10}+261894124800E_cGA\ln(2)\ln(r)r^{11}$$

$$+265531543200B_rMGA\ln(2)r^{11}+330865920000B_rr^4-433528310160E_cMr^9$$

$$-1191915648000B_r\ln(2)r^9-1099308672000E_c\ln(2)r^9+2395008000000B_r\ln(r)r^9$$

$$+175323566880B_rMGA\ln(2)r^9\Big) \qquad (24)$$

5.0 Results and Discussion

The momentum $u(r)$ and temperature $\theta(r)$ profiles were examined with various values of the physical parameters. In figure 1, the Eckert number was varied between 0.5 and 4.5,while the other parameters were kept constant that is when $p=-1,\ G=1,\ M=0.5,\ \beta=0.1\ and\ B_r=1.$ Results show that a little increase in the Eckert number slightly reduced the velocity of the fluid flow. Results further show that the velocity profiles converges to 0.5 at the point $R=2$ satisfying the boundary condition. Figure 2 shows the Grashhof number was varied between 1 and 9 while the other parameters were held constant, that is $p=-1,\ G=1,\ M=0.5,\ \beta=0.1\ and\ B_r=1.$ It is discovered that the velocity profiles converged normally to 0.5 at the point when $R=1$. Results further show that the Grashhof parameter has the tendency of slowing down the velocity of the fluid flow in the annulus of the concentric rotating cylinders.

In figure 3, the magnetic field parameter was varied between 0.5 and 2.5, with the other parameters kept constant, that is $\beta=0.1, E_c=0.5, G=1 and\ B_r=1.$ Results revealed that increase in the magnetic field increases the temperature of the system . It is further discovered that the temperature profiles converges to zero at the point where $R=2$,which shows that the wall of the outer cylinder is isothermally stable. This is due to the effects of rotation of the cylinders.

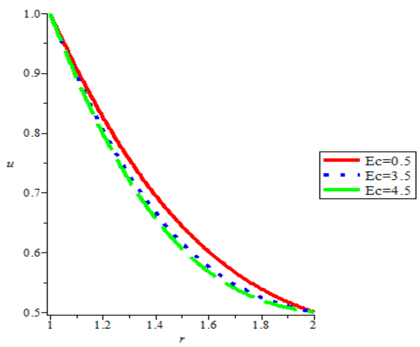

Figure 1: Velocity Profile For Various Values Of The Eckert Number

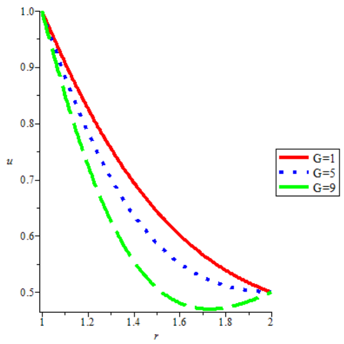

Figure 2 Velocity Profile For Various Values Of The Grashhof Number

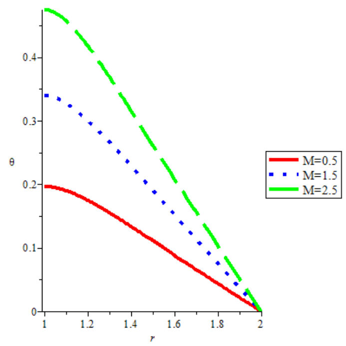

Figure 3: Temperature Profile For Various Values Of The Magnetic Parameter

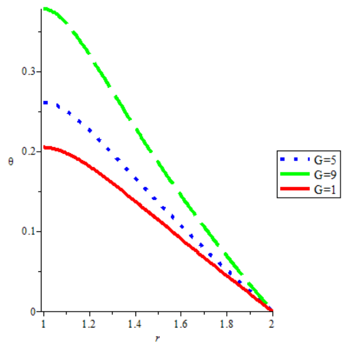

Figure 4: Temperature Profile For Various Values Of The Grashhof Number

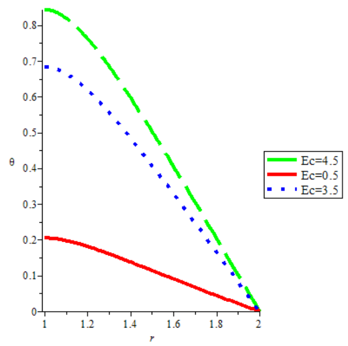

Figure 5: Temperature Profile For Various Values Of The Eckert Number

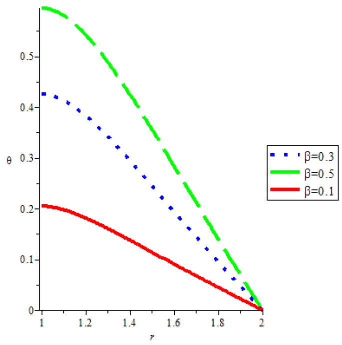

Figure 6: Temperature Profile For Various Values Of The Third Grade Parameters

In figure 4, the Grashhof number was varied between 1 and 9 and other parameters were kept constant that is β=0.1, E_c=0.5, M=0.5 and B_r=1. Results show that within the inclined rotating cylindrical annulus, that increase in Grashof number increases the temperature as the profiles converges to zero that is when $R = 2$, confirming the boundary condition, which means that the wall temperature is isothermally stable In figure 5, the Eckert number was varied between 0.5 and 4.5 in the inclined rotating cylindrical annulus when the other parameters M=0.5, β=0.1, G=1 and B_r=1 were kept constant. Results show that temperature increases with increase in the Eckert number, leading to the convergence of the profiles at zero when $R = 2$.

Figure 6 shows the third grade parameter was varied between 0.1 and 0.5 in the inclined rotating cylindrical annulus when the other parameters M=0.5, G=1 B_r=1 and E_c=0.5 were kept constant. Results show that increase in the third grade parameter increases the temperature of the system, as the profiles converges to zero when $R = 2$, which is the wall of the outer cylinder. This confirmed the boundary condition and the isothermal nature of the walls temperature of the outer cylinder.

6.0 Conclusion

In this paper, we applied perturbation technique to analyse the flow of an incompressible MHD third grade fluid through inclined rotating cylindrical pipes with isothermal wall and Joule heating. Results show that the Eckert number, the Grashhof number reduce the velocity of the fluid flow, while the magnetic field parameter, the Grashhof number, the Eckert number and the third grade parameter increase the temperature of the system. Results further show that the temperature profiles converges to zero that is when $R = 2$, confirming the boundary condition, which means that the wall temperature is isothermally stable.

REFERENCES

Aiyesimi, Y.M., Okedayo, G.T., and Lawal,O.W. (2014).Effects of magnetic field of the MHD flow of a third grade fluid through inclined channel with Ohmic heating.
Journal of Applied and computational Mathematics .

Ayub, M., Rasheed, A., and Hayat, T.(2003): Exact flow of a third –grade fluid past a porous plate using homotopy analysis method. Int. J. Eng. Sci. 41(2091).

Baldoni, F., Gudhe, R. and Yaleswarapu, K.K. (1993): Helical flow of a simple fluid between eccentric cylinders. Int. J. Non-linear Mech. 28, 221-235.

Erickson, J.L. (1956): Over-determination of the speed in rectilinear motion of non-Newtonian fluid. Q. Appl. Math.14, 318-321.

Feiz-Dizali, A., Salimpour, M.R. and Jam, F. (2007). Flow field of a third grade non- Newtonian fluid in the annulus of rotating concentric cylinders in the presence of magnetic field. Elsevier03/110.

Fosdick R.L. and Rajagopal, K.R. (1980): Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 339, 351-377.

Fosdick, R.L. and Kao, B.G. (1978). Tranverse deformations associated with rectilinear shear in elastic solid. J. Elasticity, 8, 117-142.magnetic field. Elsevier03/110.

Hayat, T., Nadeem, S., Asghar, S. and Siddiqui, A.M. (2001): MHD rotating flow of third grade fluid on an oscillating porous plate. Acta Mechanica 152, Issue1-4, 177- 190.

Makinde, O.D.(2005): Strong exothermic explosions in a cylindrical pipe: Acase study of series summation technique. Mech. Res. Common ., 32, 191-195.

Makinde, O.D. (2007): Thermal stability of a reactive third grade fluid in a cylindrical: of Hermite-Pade approximation technique. Elsevier Science Direct. Applied Mathematics and Computation, 189,S 690-697.

Massoudi, M. and Christie, I. (1995): Effects of variable viscosity and viscous dissipation on the flow of a third –grade flu An exploitation id in a pipe. Int. J. of Nonlinear Mech., 30(5): 687-699.

Massoudi, M., Vaidya, A. and Wulandana, R. (2008): Natural convection flow of ageneralized second grade fluid between two vertical walls. Nonlinear Analysis. Real World Applications 9, 89-93

Obi, B.I., Okedayo G.T., Jiya, M. Aiyesimi, Y.M. (2017). Annular flow of an incompressible MHD third grade fluid in a rotating concentric cylinders with isothermal walls and Joule heating. NJTR. 12(2):34-42.

Rajagopal, K.R. (1980): On the stability of third grade fluids. Arch. Mech. 32, 867-875.

Rajagopal, K.R. (1993): Mechanics of non-Newtonian fluids in G.P. Galdi. J. Necas(Eds). Recent Development in Theoretical Fluid Mechanics, Pitman Research Notes in Mathematical Series, 291, Longman Scientific and Technical, New York.

Rajagopal, K.R., Szeri, A.Z. and Troy, W. (1986). An existence theorem for the flow of non-Newtonian fluid past an infinite porous plate. Int. J. Non-linear Mech. 21, 279-289.

Rajagopal, K.R. and Mollica, F. (1999). Secondary deformation due to axial shearing of annular region between two eccentrically placed cylinders. Int. J. Engng. Sci 37, 411-429

Szeri, A.Z. and Rajagopal, K.R.(1998). Flow of a non-Newtonian fluid between heated parallel plates. Int. J. of inear Mech. 20(2): 91-101.

Yurusoy, M. and Pakdemirli,M. (2002). Approximate analytical solution for the flow of a third-grade fluid in a pipe. Int. J. of inear Mech.,37:187-195.