Behaviour of shear stress of plate girders with corrugated webs

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ABSTRACT

This paper presents analytical studies using Minimum Potential Energy Method to determine critical shear stress of local and global buckling of plate girder with corrugated webs. The elastic interactive shear buckling stress of corrugated steel web is calculated by all possible failure criteria (steel yielding, local and global buckling stresses). The results are compared with Finite element method (FEM) using ANSYS. It found that the proposed equations are a good agreement with the results FE.

Keywords: Corrugated webs, Shear buckling, minimum potential energy, ANSYS.

1 Introduction

Most of studies focused on steel girders with corrugated webs were about the shear and bending behaviour of simply supported beams.

Elgaaly [1] carried out experimental and numerical analysis on trapezoidal web plate girder subjected to shear load. He developed a formula based on Timoshenko and Gere [2] local buckling strength as shown in Eq. [1]. In the local buckling mode, the trapezoidal webs act as a series of flat sub-panels [3], that mutually supported each other along their vertical (longer edges) and are supported by the flanges at their horizontal (shorter) edges. The elastic local buckling stress, τ_l as proposed by Elgaaly is expressed as:

$$\tau_{crl} = K_s \frac{\pi^2 E}{(1-\nu^2)}\left(\frac{t_w}{z}\right)^2 \qquad [1]$$

Where: $K_s = 5.34 + 4\left(\frac{z}{h_w}\right)^2$ For all edges simply supported

$K_s = 8.98 + 5.6\left(\frac{z}{h_w}\right)^2$ For all edges clamped

$K_s = 5.34 - 2.31\left(\frac{z}{h_w}\right) - 3.44\left(\frac{z}{h_w}\right)^2 + 8.98\left(\frac{z}{h_w}\right)^3$ For long edges simply supported

From several studies govern; the local buckling failure stress is expressed as:

$$\tau_{cr,l} = 1.41\frac{\left(\frac{t_w}{b}\right)\sqrt{E f_y}}{\gamma_m} \leq \tau_y \qquad [2]$$

Where: γ_m=1.10 for buckling failures and 1.0 failure by yielding, they proposed Eq.[2] for the design local buckling stress

The new empirical equation for critical shear stress of a trapezoidal web plate proposed by Fathoni Usman [4] is written as:

$$\tau_l = k\frac{1.6\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t_w}{b}\right)^2 \quad [3]$$

$$where: k = \frac{1.8}{\left(\frac{b}{h_w}\right)^2} - \left(\frac{b}{h_w}\right)^3 + 8\left(\frac{b}{h_w}\right) + 9$$

Easley [5] calculate the elastic critical global buckling shear stress (τ_{crg}):

$$\tau_{crg} = 36\gamma\frac{D_x^{0.25}D_y^{0.75}}{h_w^2 t_w} \quad [4]$$

Where γ =factor to represent different end restraint condition ($1 \leq \gamma \leq 1.9$)

Hlavacek's [6] formula was analyzed by Easley and was reduced to:

$$\tau_{crg} = 41\frac{D_x^{0.25}D_y^{0.75}}{h_w^2 t_w} \quad [5]$$

Also The global elastic buckling stress calculated by Galambos [7]:

$$\tau_{crg} = k_g\frac{D_x^{0.25}D_y^{0.75}}{h_w^2 t_w} \quad [6]$$

Where: kg = 36 for steel flanges and = 68.4 for composite flanges (Elgaaly [1]).

Yi et al. [8] 2008 provide an alternate expression for τ_{crg} as follows:

$$\tau_{crg} = k_g\frac{\pi^2 E}{12(1-\upsilon^2)\left(\frac{h_w}{t_w}\right)^2} \quad [7]$$

$$k_g = 5.72\left(\frac{h_r}{t_w}\right)^{\frac{3}{2}} \qquad \text{Where } h_r = c.\sin\alpha$$

2 Theoretical Analysis

The theoretical analyses by using The Minimum Potential Energy Method to predict the critical shear stress of local and global shear buckling.

2-1 Calculation of local shear buckling

Corrugated web in local buckling mode of failure acts as a series of flat plate sub panels (isotropic plate) some research assumed that the web is mutually support each other along their vertical (longer) edges and are supported by the flanges at their horizontal (shorter) edges. These flat plate sub panels are subjected to shear, the elastic buckling stress considering this plate as isotropic plates, in this research we choose (Hamed, A.Y [9]), that the web is simply supported each other along shorter edges(X-axis)direction. Whereas the other two edges (Y-axis) direction, at the flange junction tend to be fixed due to the large torsional stiffness in the case of laterally supported compression flanges, where the upper flange (subjected to compressive stress) is fully braced by a concrete slab whereas lower flange (subjected to tension stress) is also laterally braced.

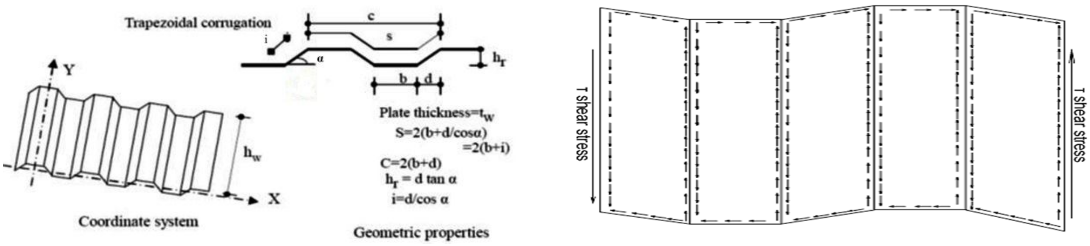


Fig. [1]: Dimension of corrugation profile and distribution of shear stress

2-2-1 Estimation of local buckling coefficient

The method of The Minimum Potential Energy depends on assuming a suitable deflection form *w* which satisfies the assumed end conditions.

$$w = A_1 \sin\frac{\pi}{b}x\ \sin^2\frac{\pi}{h_w}y + A_2\ \sin\frac{2\pi}{b}x\ \left(\cos\frac{\pi}{h_w}y - \cos\frac{3\pi}{h_w}y\right) \qquad [8]$$

Where: A_1, A_2 are the unknown coefficients.

The stored strain energy is given by the equation:

$$U_B = \frac{D}{2}\int_0^b\int_0^{h_w}\left[\left(\frac{\delta^2 w}{\delta x^2}+\frac{\delta^2 w}{\delta y^2}\right) - 2(1-\upsilon)\left(\frac{\delta^2 w}{\delta x^2}\frac{\delta^2 w}{\delta y^2} - \left(\frac{\delta^2 w}{\delta x \delta y}\right)^2\right)\right]dxdy$$

After substituting with *w* and performing. The required integrations, we get:

$$U_B = \frac{D}{2}(C_1 A_1^2 + C_2 A_2^2) \qquad [9]$$

Where:

$$C_1 = \left(\frac{3}{16}\frac{\pi}{z^3}^4 h_w + b\frac{\pi}{h_w^3}^4 + \frac{1}{2z}\frac{\pi}{h_w}^4\right)$$

$$C_2 = \left(8\frac{\pi^4 h_w}{z^3} + \frac{41}{2}\frac{\pi^4 z}{h_w^3} + 20\frac{\pi^4}{z\,h_w}\right)$$

$$\mathrm{T} = \tau_{xy}\mathrm{t}\int_0^{\mathrm{b}}\int_0^{\mathrm{h_w}}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\frac{\partial \mathrm{w}}{\partial \mathrm{y}}\,\mathrm{dxdy} \qquad [10]$$

After substitution:

$$T = -\frac{256}{45}\tau_{av} t_w A_1 A_2 \qquad [11]$$

After the calculation of U and T the total potential energy V is obtained from which the two equations [Eq.9, 11] are derived. These results are two simultaneous linear equations in the unknown parameters A_1 and A_2 and containing values for τ.

$$V = \frac{D}{2}(C_1 A_1^2 + C_2 A_2^2) - \frac{256}{45}\tau_{av} t A_1 A_2 \qquad [12]$$

Probably the best known energy technique is the Rayleigh-Ritz procedure which requires that:

$$\frac{\partial V}{\partial A_1} = 0 \quad , \quad \frac{\partial V}{\partial A_2} = 0$$

$$\tau_{crL} = \frac{45}{256t} D \frac{\pi^4}{h_w^2} \sqrt{\psi_{L1}\psi_{L2}} \qquad [13]$$

When $D = \frac{Et^3}{12(1-\upsilon^2)}$, as isotropic plate, we arrive to the following relationship:

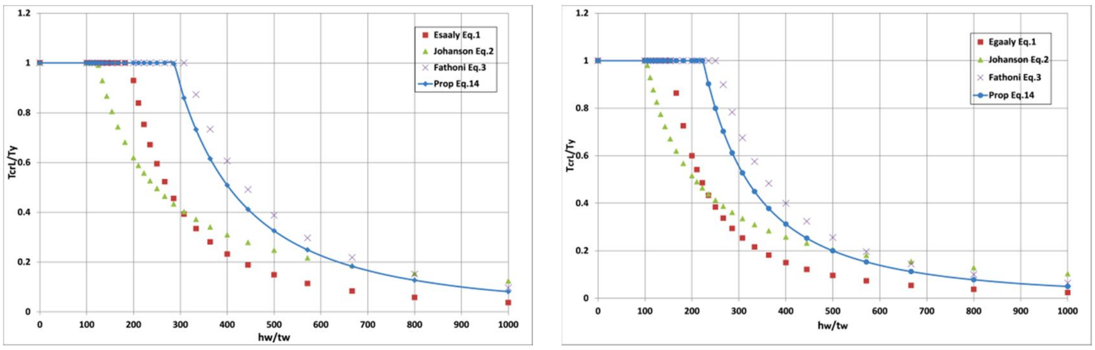
$$\tau_{crL} = \frac{45\pi^2}{256} \sqrt{\psi_{L1}\psi_{L2}} \frac{E\pi^2}{12(1-\upsilon^2)} \left(\frac{t_w}{z}\right)^2 \qquad [14]$$

Where: $k_L = \frac{45\pi^2}{256} \sqrt{\psi_{L1}\psi_{L2}}$

$$\psi_{L1} = \frac{1}{2}\left[\frac{3}{8}\left(\frac{h_w}{z}\right) + 2\left(\frac{z}{h_w}\right)^3 + \frac{1}{2}\left(\frac{z}{h_w}\right)\right] \qquad [15a]$$

$$\psi_{L2} = \frac{1}{2}\left[16\left(\frac{h_w}{z}\right) + 41\left(\frac{z}{h_w}\right)^3 + 40\left(\frac{z}{h_w}\right)\right] \qquad [15b]$$

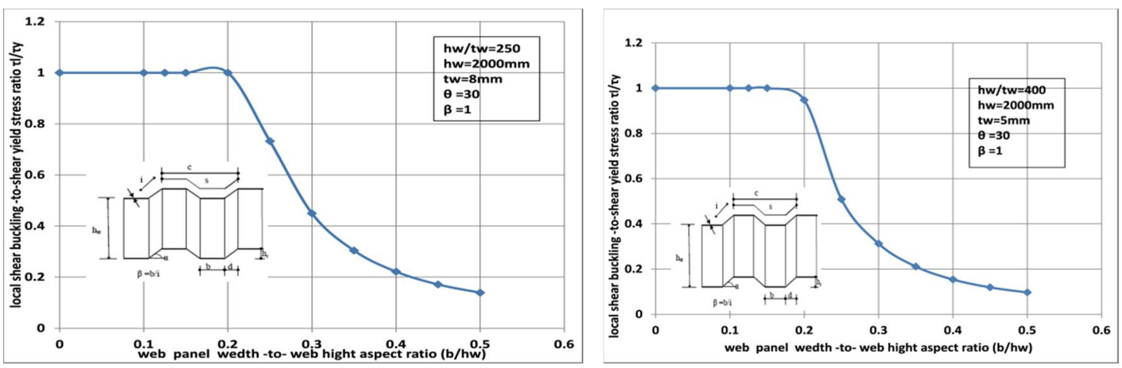
[Eq.14] which is the final required equation which links the critical shear stress in the case of local buckling subjected to shear stress with the prescribed boundary conditions. Comparison the result with other research, [Eq.1], [Eq.2], and [Eq.3]. As see fig. [2].



b/h_w=0.25 b/h_w=0.3

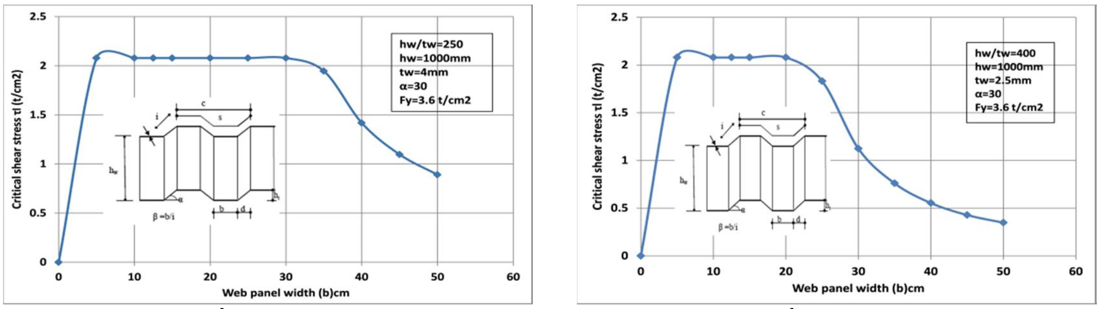
Fig. [2]: Comparison of local stress Eq.(14) with other researchers

2-2-2 Parametric study of critical shear failure mode



$(\frac{h_w}{t_w} = 250)$ $(\frac{h_w}{t_w} = 400)$

Fig. [3]: Effect of $\frac{b}{h_w}$ on shear buckling modes for trapezoidal corrugated web.



$(\frac{h_w}{t_w} = 250)$ $(\frac{h_w}{t_w} = 400)$

Fig. [4]: Critical shear stresses for girders with trapezoidal corrugated webs.

3-2 Calculation of global shear buckling

The global shear buckling stress of the corrugated webs treated the corrugated web as an orthotropic flat plate. We recall that the assumption of isotropy implies that material properties at a point are the same in all directions. This means, that if an isotropic material is subjected to an axial stress in a principal direction, the major deformation occurs in the direction of the applied load. Lateral deformations of smaller magnitude occur in the other principal directions. Also, shear stress causes only shear deformation. So, normal strains and stresses are not coupled to shear strains and stresses. The deformations are dependent on the two independent elastic constants for instance, E and υ.

3-2-1 Estimation of global buckling coefficient

Assuming a suitable deflection form *w* Eq.[8] which satisfies the assumed end conditions. The expression for the potential energy of bending for orthotropic plates, which follows from:

$$U = \frac{1}{2}\int_0^b \int_0^{h_w} \left[D_x\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{xy}\left(\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right) + D_y\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_s\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dxdy \qquad [16]$$

Where: D_x; D_y; D_{xy}; D_{yx} and D_s are the flexural and torsional rigidities of an orthotropic plate.

$$D_x = \frac{E t_w^3\left[\left(\frac{d}{t_w}\right)^2 + 1\right]}{6\eta} \qquad [17a]$$

$$D_y = \frac{E t_w^3}{12(1-\nu^2)}\eta \qquad [17b]$$

$$D_s = G\frac{t^3}{12} \qquad [17c]$$

$$D_{xy} = D_y \qquad [17d]$$

$$\eta = \frac{a+b}{a+l} \qquad [17e]$$

After substituting with *w* and performing the required integrations, we get the final equation that relates shear stress for orthotropic corrugated web plate:

$$\tau_{crg} = \frac{45}{256t}\frac{\pi^4}{h_w^2}\sqrt{\psi_{g1}\psi_{g2}} \qquad [18]$$

Where:

$$\psi_{g1} = \frac{1}{2}\left[D_x\left(\frac{3}{8}\left(\frac{h_w}{a}\right)^3\right) + 2H\left(\frac{1}{2}\frac{h_w}{a}\right) + D_y\left(2\frac{a}{h_w}\right)\right] \quad [19a]$$

$$\psi_{g2} = \frac{1}{2}\left[D_x\left(16\left(\frac{h_w}{a}\right)^3\right) + 2H\left(20\frac{h_w}{a}\right) + D_y\left(41\frac{a}{h_w}\right)\right] \quad [19b]$$

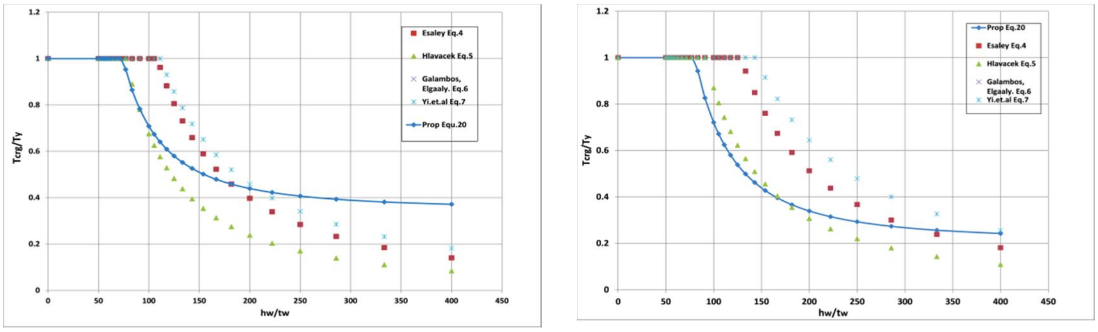
Also we can write the equation [Eq.18] at the form:

$$\tau_{crG} = \frac{45\pi^4}{512}\mu_g \frac{D_x^{\frac{3}{4}} D_y^{\frac{1}{4}}}{t h_w^2} \quad [20]$$

Where: $\mu_g = \sqrt{\left[\begin{array}{c} 6\sqrt{\frac{D_x}{D_y}}\left(\frac{h_w}{a}\right)^3 + 31\left(\frac{h_w}{a}\right)^4 + \frac{699}{8}\sqrt{\frac{D_y}{D_x}}\left(\frac{h_w}{a}\right)^2 + 49\frac{D_y}{D_x} \\ +82\sqrt{\left(\frac{D_y}{D_x}\right)^3}\left(\frac{a}{h_w}\right)^2 \end{array}\right]}$

Where, a: The distance between the load and the support

Eq.[20] which is the final required equation which links the critical shear stress in the case of global buckling subjected to shear stress with the prescribed boundary conditions, comparison the result with other research [Eq.4], [Eq.5] , [Eq.6] and [Eq.7]. As see fig. [5].



b/h_w=0.15 b/h_w=0.10

Fig. [5]: Comparison of global stress Eq.[20] with other researchers

3-2-2 Parametric study of shear failure

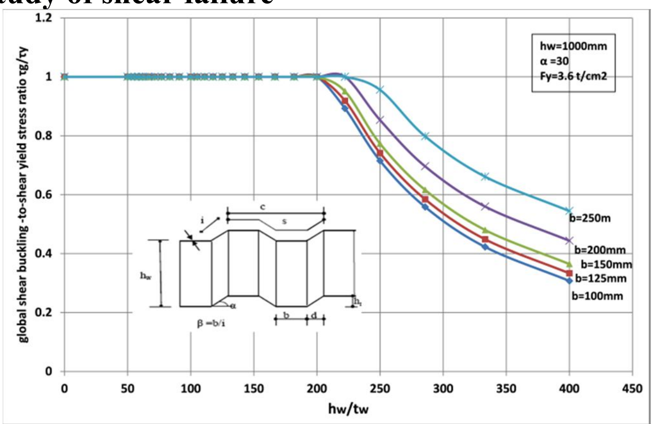


Fig. [6]: Effect of breadth of panel (b) on critical shear stress for trapezoidal corrugated web.

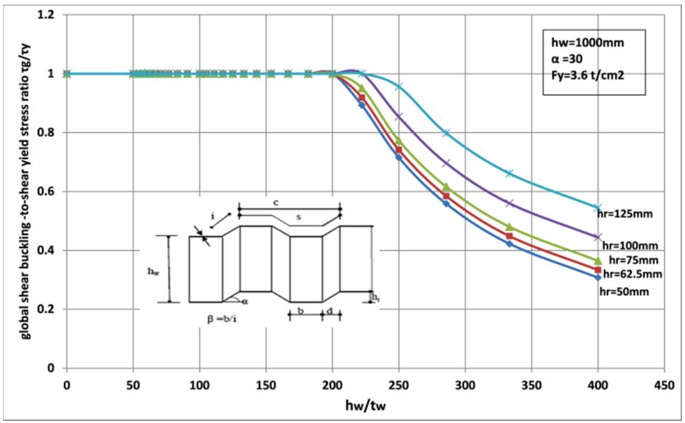


Fig. [7]: Effect of height of corrugated (h_r) on global shear buckling.

2-2 Interactive shear buckling

The elastic interactive shear buckling stress of corrugated steel web is given by, this includes all possible failure criteria (steel yielding, local and global buckling stresses) which is given by equation [21] by El-Metwally and Loov [10] as follows:

$$\frac{1}{\tau_{cr,i}^{n}} = \frac{1}{\tau_{cr,l}^{n}} + \frac{1}{\tau_{cr,g}^{n}} + \frac{1}{\tau_{y}^{n}} \qquad [21]$$

Considering two values of exponent "n" equal to 3 and 5 to depict which one of them is better for the girders. It was cleared that modified equation results are nearly simulate the results obtained from finite element in case of "n" equal to 5 study by the author [11]. Therefore modified equation can be used with good accuracy to study the behaviour of plate girders with corrugated steel webs under shear.

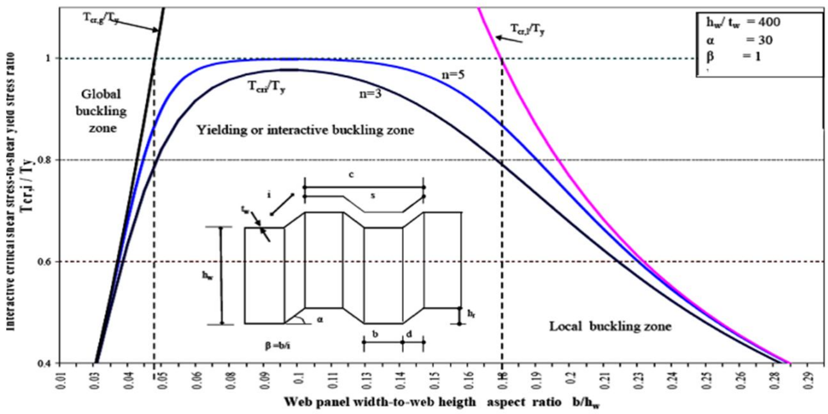


Fig. [8]: Effect of b/h_w on shear buckling modes for trapezoidal corrugated web.

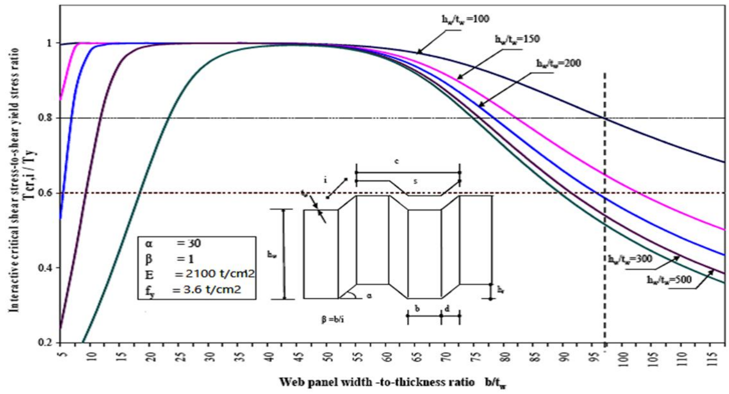


Fig. [9]: Effect of b/t_w on interactive shear buckling modes for trapezoidal corrugated web.

3 Verification of proposed design equations and Finite element

In this section, the proposed shear stress of corrugated steel webs is verified using the Energy Method (Minimum potential energy). Proposed shear strengths are also compared with those of FE (ANSYS 12) [13].

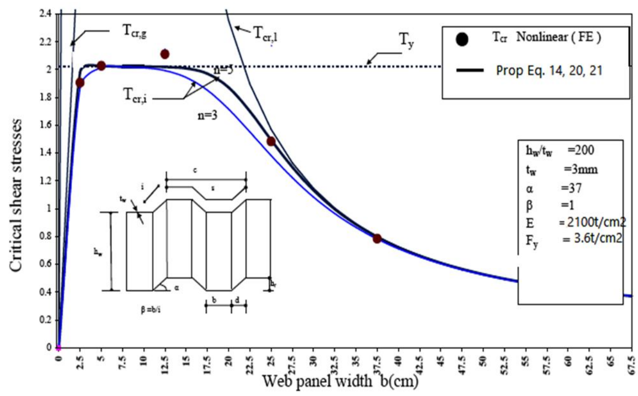


Fig. [10]: Verification of proposed design equations and Finite element

4 Conclusions

The study presented in this research which is, mainly, based on an accurate theoretical procedure to have the critical shear stress of the various mode of buckling (local, global and interactive buckling) of trapezoidal corrugated steel webs, Using the minimum potential energy in order to verify the proposed equations, supported by a finite element check for the behaviour of the plate girder with corrugated web under pure shear, can be used to obtain a variety of results. The careful inspection of these results gives the following conclusions:

1- A significant effect on the global and local buckling in case of dense and coarse corrugation respectively, while in case of moderate value of $\frac{b}{h_w}$ it has small effect on the interactive buckling mode which may be close to shear yield stress for a wide range of $\frac{b}{h_w}$. It was recommended that the aspect ratio $\frac{b}{h_w}$ value should be in the range between (0.05-0.375) to have economic sections.

2- It has a significant effect on interactive shear buckling in global buckling zone effect. In local buckling zone it has a clear effect on interactive shear buckling in case of small values of $\frac{h_w}{t_w}$, but in case of larger values of $\frac{h_w}{t_w}$ it has negligible effect. In general increase of $\frac{h_w}{t_w}$ leads to decrease shear stress of corrugated web.

Notation

a	Distance between the stiffener and the support.
b	Width of horizontal fold of corrugation.
c	Horizontal length of one corrugation
hw	Web height.
h_r	Corrugated panel height
i	Width of inclined fold of corrugation.
α	Angle of corrugation.
E	Modulus of elasticity (2100 t/cm^2)
F_y	Yield stress of web material.
z	Width of horizontal or inclined panel.
τ_y	Shear yield stress of web material.
τ_l	Local shear buckling.
τ_g	Global shear buckling
τ_i	Interactive shear buckling.
$\frac{b}{h_w}$	Aspect ratio; web panel width-to-web height ratio.
υ	Poisson's ratio.

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