APPLICATION OF ARIMA MODELLING
(A Case Study of Stock Market Prices of Julius Berger Nigeria Limited)

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ABSTRACT
This study examined an Autoregressive Integrated Moving Average (ARIMA) model for the average stock market prices (in Naira) of Julius Berger Nigeria PLC, for the period of January 2006 to December 2012 obtained from the website of cash craft asset management limited (a member of Nigerian Stock Exchange), while the 2013 Figures were used to assess the forecasting performance of the fitted model. The result of the analysis using the MINITAB software package version 15.0 revealed that the data required first order differencing to isolate trend. Further analysis showed that the appropriate model that best described the pattern is the Seasonal Autoregressive Integrated Moving Average process of order $(1,1,1)x(1,0,1)_{12}$.

Key words: ARIMA, Stock Market Prices, Bartletts test, ACF, PACF

Introduction

Stock exchange is a mutual organization that facilitates to the stock brokers and traders (Edo; 1995). The stock exchange deals with issues of securities and other financial instruments. The stock exchange is also involved in the payment of income and dividends. The securities that are traded in the stock exchange are shares, unit trusts and bonds. There are two kinds of stock markets, the physical market and virtual market. In the physical market, trading takes place through open outcry system while in the virtual market trading is through the electronic network (Edo; 1995). The initial public offering of stock and bonds take place in the primary markets, although the trading that follows takes place in the secondary market.

The stock market plays a major role in financial intermediation in both developed and developing countries by channeling idle funds from surplus to deficit units in the economy. As the economy develops, more resources are needed to meet needed rapid expansion; the stock market serves as a veritable tool in the mobilization and allocation of savings among competing uses which are critical to the growth and efficiency of the economy.

Statement of Problem

Over the years, it has been believed that stock market impact positively on economic growth but little information have been provided empirically to show the direction and magnitude of this impact especially in Nigeria. Given the number of years, since the Nigerian stock market has been established and the substantial financial resources available in the country, coupled with the existing institutions, one can claim that the entire spectrum of the capital market has not been sufficiently active, especially when compare with the capital unit of similar or lesser aged units in other developing countries. The problem includes high cost of transaction and lack of communication. Over the years, the market capitalization in the floor of Nigeria Stock Exchange has been on the increase but corresponding growth of GDP has not been witnessed within the period. For instance, the market capitalization rose from 215 billion in 1981 to 397.9 billion in 1982 which signifies about 85% growth and it continued to rise through 1986 by average of 23% while the decade beginning from 1980 witnessed a rapid fall of growth rate of GDP. It is against this background that this study would be undertaking to examine the stock market prices at the Nigeria Stock exchange using Julius Berger Nigeria Plc as the case study.

Related Literature Review

There is need to review other people's research work, which in one way or the other relates to this research.

Okafor (2013) carried out a research on Time series analysis of stock prices at the Nigerian stock exchange with first bank of Nigeria as the case study from 2006 to 2010. The additive model was adopted for the decomposition of the time series component; because the plot of the yearly seasonal standard deviation shows no appreciable increase or decrease to the season means. The bartletts test of homogeneity of variance showed that the data followed a constant variance. The time series estimations, such isolation of the trend, isolation of the seasonal effect, isolation of the cyclical component and residual test were properly carried out. The normality test showed that the data do not follow a normal distribution. The result from the analysis showed that there was depreciation in growth of the stock prices of first bank of Nigeria during the years under study.

Iwueze et al (2013) modelled the Nigeria External Reserve using ARIMA from January 1999 to December 2008. Results of the analysis showed that the data required logarithmic transformation to stabilize the variance and make the distribution normal. Again, the result revealed that the appropriate model that best described the pattern in the transformed data was the Autoregressive-Integrated Moving Average process of order (2,1,0).

Okafor and Shaibu (2013) used the ARIMA model to study Nigerian inflation Dynamics, using a yearly time series data from 1981 to 2010. The study used the Ordinary Least Squares (OLS) technique for estimation purposes. On the basis of various diagnostic and selection evaluation criteria, the best model was selected for the short term forecasting of Nigerian inflation. The study found ARIMA (2,2,3) as the most appropriate model under model identification, parameter estimation, diagnostic checking and forecasting inflation. In- sample forecasting was attempted and the estimated ARIMA model remarkably tracked the actual inflation during the sample period. The study concluded that Nigerian inflation is largely expectations-driven. The major inference that can be drawn was that expectations that were formed about future levels of prices affect the current purchase decisions.

Ayinde and Abdulwahab (2013) used the multiplicative SARIMA to study the Nigeria crude oil exportation from January 2002 to December 2011. Result revealed an upward trend of the series which became stationary at 1st difference, a sharp drop between 2007 and 2009 and autocorrelation function with significant spikes at lag 1, 7 and 12 suggesting the presence of seasonality in the series. Based on Akaike Information Criterion (AIC), Schwartz Bayesian Information Criterion (SBIC) and Hannan-Quinn Information Criterion (HQC), the best model was SARIMA (1,1,1) x $(0,1,1)_{12}$. The diagnosis on such model was confirmed, the error was white noise, presence of no serial correlation and a forecast for current and future values within 24 months was made which indicated that the crude oil exportation was fairly unstable.

Methodology

Data Source and Limitations

The data used for this study is a secondary data collected from the website of cash craft asset management limited (a member of Nigerian Stock Exchange). The data is on monthly average stock market price (₦) of Julius Berger Nigeria Plc, from 2006 to 2012. The data were compiled from daily stock market prices, to weekly stock market prices and finally monthly stock market prices. The limitation of this data is that its arrangement was difficult, since it came in bulk form, which was in daily prices, weekly and finally on monthly stock prices.

Method of Analysis

The statistical technique adopted in this study is the Box-Jenkins (1976) iterative procedure for fitting Autoregressive Integrated Moving Average (ARIMA) model. The Box-Jenkins model for stationary time series data are the Autoregressive process of order 1 [AR(1)], Autoregressive process of order 2 [AR(2)], Moving Average process of order 1 [MA(1)], Moving Average process of order 2 [MA(2)] and Autoregressive Moving Average of order (1,1), [ARMA (1,1)].

The models are;

For AR (1):

$$X_t = \phi_0 + \phi_1 X_{t-1} + e_t \quad (1)$$

For AR(2):

$$X_t = \phi_0 + \phi_1 X_{t-1} + \phi_2 X_{t-2} + e_t \quad (2)$$

Where $\phi_0, \phi_1, \phi_2,$ are constants, and e_t is a white noise process

For MA(1):

$$X_t = \theta_0 + \theta_1 e_{t-1} + e_t \ . \quad (3)$$

For MA(2):

$$X_t = \theta_0 + \theta_1 e_{t-1} + \theta_2 e_{t-2} + e_t \quad (4)$$

Where $\theta_0, \theta_1, \theta_2,$ are constants

For ARMA (1, 1), the model is

$$X_t = \mu + \phi_1 e_{t-1} + \theta_1 e_{t-1} + e_t \quad (5)$$

Where μ, ϕ_1, θ_1 are constants.

The Box-Jenkins iterative procedure for fitting time series model consist of Model identification, parameter estimation, and diagnostic checking

(i) Model Identification; Autocorrelation function (ACF)

The appropriate model is said to be

AR (1) if the partial autocorrelation function (PACF) cuts off after lag 1,

AR (2), if the PACF cuts off after lag 2

MA (1) if the ACF cuts off after lag 1

MA (2) if the ACF cuts off after lag 2

ARMA (1, 1) if the ACF and PACF cut off after lag 1

(ii) Parameter Estimation: The estimate of parameters of this selected model was obtained using MINITAB software

(iii) Diagnostic Checking: The properties of the residuals e_t are what was used for diagnostic checking. After fitting the tentative model to a study data, the residuals from the fitted model are subjected to what is often referred to as residual analysis. These include checking if the;

(i) Mean of e_t $[E(e_t)]$ is zero

(ii) The variance (σ^2) is constant and,

(iii) ρ_k and ϕ_{kk} are zero for all k.

The fitted model is said to be adequate for a study data if the residual have these properties.

For non-stationarity arising from polynomial trend, non-seasonal differencing of order d (d = 0,1,2) is required to isolate the trend. For non-stationarity arising from seasonal effect, seasonal differencing of order D (D = 0,1,2) is required1 to isolate the seasonal effect.

The large lag standard error required to test the null hypotheses,

$H_0 : \rho_k = 0 \quad k>q \text{ Vs } H_1 : \rho_k \neq 0$

$k > q$ is given by

$$\sigma = \sqrt{\frac{1}{n}(1 + 2\sum_{i=1}^{q} r_k^{\ 2})} \tag{6}$$

The Box-Jenkins model assumes that the underlying distribution of the study data is normal with constant variance.

To test for normality, the Anderson-Darling test is used. Under the null hypothesis,

H_0: Data is normally distributed

H_1: Data do not follow normal distribution.

The Anderson-Darling test statistic is given by

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln F(X_i) + \ln(1 - F(X_{n-i+1}))] \tag{7}$$

Where

n is the number of observations

F is the cumulative distribution function of the normal distribution

To test for constant variance, the Bartlett's test is used.

To test the null hypothesis:

$$H_0 : \sigma_1^{\,2} = \sigma_2^{\,2} = \sigma_3^{\,2} = \ldots = \sigma_n^{\,2} = \sigma^2$$

$$H_1 : \sigma_i^{\,2} \neq \sigma_j^{\,2}$$

Bartlett test statistic is given as;

$$\chi_0^2 = \frac{N - K\ln(S_p^{\,2}) - \sum_{i=1}^{k}(n_i - 1)\ln(S_i)^2}{1 + \frac{1}{3(k-1)}\left(\sum\left(\frac{1}{n_i - 1}\right) - \frac{1}{N-K}\right)} \tag{8}$$

Where

$$S_p^{\,2} = \frac{\sum_{i=1}^{k}(n_i - 1)S_i}{N - K} \tag{9}$$

$S_p^{\,2}$ is the pooled variance

Ni is the number of sample from the ith month

Si is the variance of the ith month

N is the total number of sample

K is the total number of month.

The test statistic is approximately a $\chi^2_{k-1,\alpha}$ (chi-square distribution with k-1 degree of freedom)

We accept the null hypothesis if $\chi_0^2 < \chi^2_{k-1,\alpha}$

Forecasting

One of the objectives of model building is to provide forecast of the future values. In producing the forecast using the fitted model, it is assumed that the condition(s) under which the model was constructed would persist in the periods of the forecast are made. Using the ARIMA model, the MINITAB software will be used to get the forecast.

Data Analysis

In this section, the methodology discussed in this study shall be implemented to analyze the data and conclusions shall also be drawn based on the results obtained from the analysis. In this research, the analysis was carried out via MINITAB version 15.0 statistical software and Microsoft excel 2007 version.

Data Evaluation

The monthly record of Average stock market price of Julius Berger Nigeria Plc from 2006 to 2012 is used in this study.

Table 1: Annual and overall means and standard deviation of stock market price of Julius Berger plc.

Year	Mean	Std

2006	3.510	0.221
2007	6.847	1.372
2008	13.093	2.340
2009	4.733	1.585
2010	6.492	2.175
2011	10.015	1.060
2012	3.758	1.029
Overall mean	6.921	
Overall Std		0.726

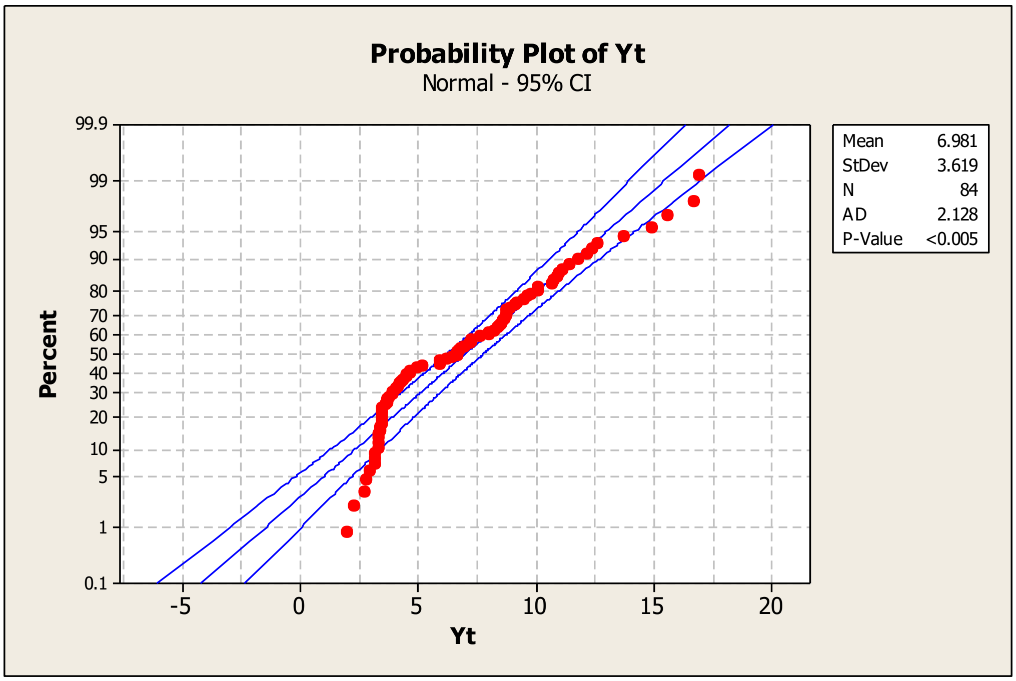

Fig.1: Probability plot of the series Y_t

Table 2: Descriptive Statistics Values

N	N^*	Mean	Std. Dev	Median	Minimum	Maximum	Skewness	Kurtosis
84	0	6.981	3.619	6.695	2.030	16.90	0.78	- 0.03

From the MINITAB output of the Anderson-Darling goodness of fit test, it shows that the series Y_t is not normally distributed. However, since the data size is large (84), we assume the central limit theorem.

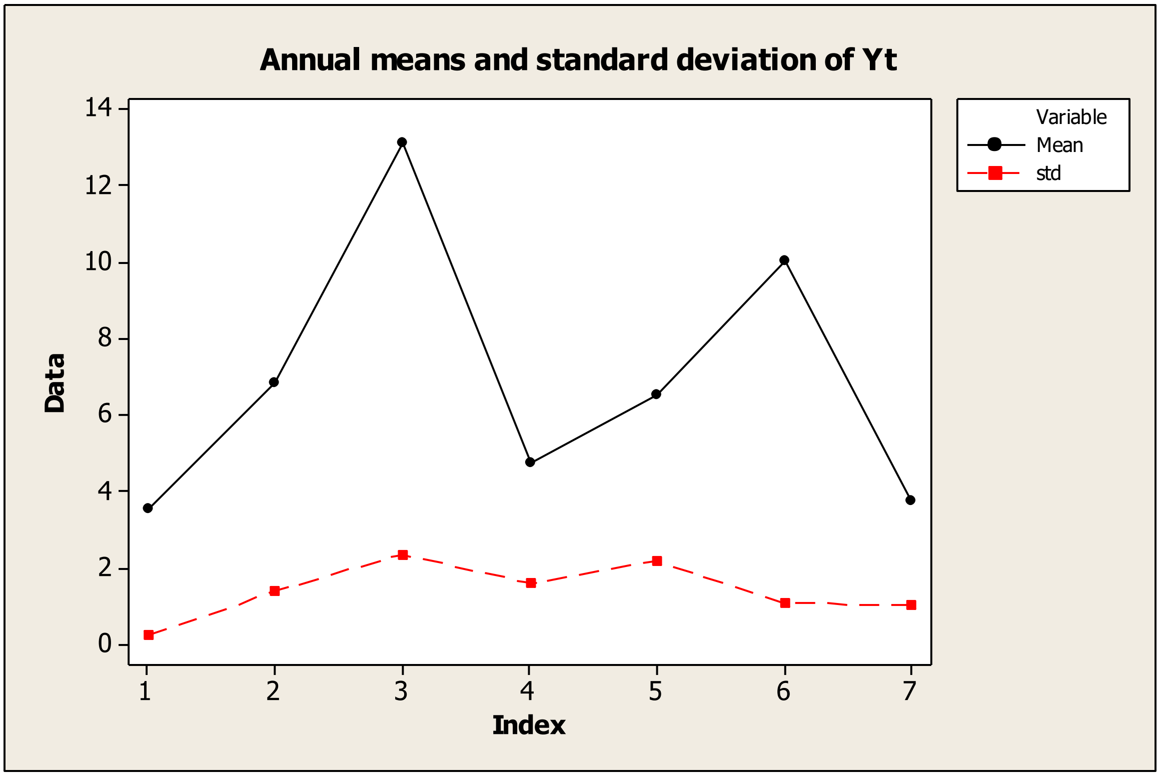

Fig. 2: Annual means and standard deviation of the series Y_t

The plot of the annual means and standard deviation shown in Fig. 2 and Table 1 indicates that the standard deviation is not stable implying that the variance may not be constant.

To confirm the authenticity of the statement above, we use the Bartlett test statistic, to test the hypothesis. The Buys-Ballot table in this work produces the monthly variance and their natural logarithms as presented in Table 3.

Table 3: Monthly variance and their natural logarithms of the series Y_t

(s_i^2)	8.743	11.209	21.688	20.821	16.712	15.721	12.062	9.986	10.446	9.697	7.268	7.339
$\ln s_i^2$	2.168	2.417	3.077	3.036	2.816	2.755	2.490	2.301	2.346	2.272	1.984	1.993

The pooled variance $S^2{}_p$ from Equation (9) gives

$$S^2{}_p = \frac{\sum_{i=1}^{7}(7-1)s_i^2}{84-12}$$

$$= \frac{6\sum_{i=1}^{7}s_i^2}{84-12} = \frac{910.152}{72} = 12.641$$

$$\chi_0^2 = \frac{4.730}{1.035} = 4.570$$

$$\chi^2_{k-1,\alpha} = \chi^2_{6,0.05} = 12.592$$

$\chi_0^2 = 4.570 < \chi_{k-1,\alpha}^2 = 12.592$

Therefore we conclude that there is no significant difference in the monthly variances which means that the data has constant variance.

Model Identification, Parameter Estimation and Diagnostic Checking

The time series plot is shown in Figure 3. From the plot, the series appears to be oscillating, around a horizontal line

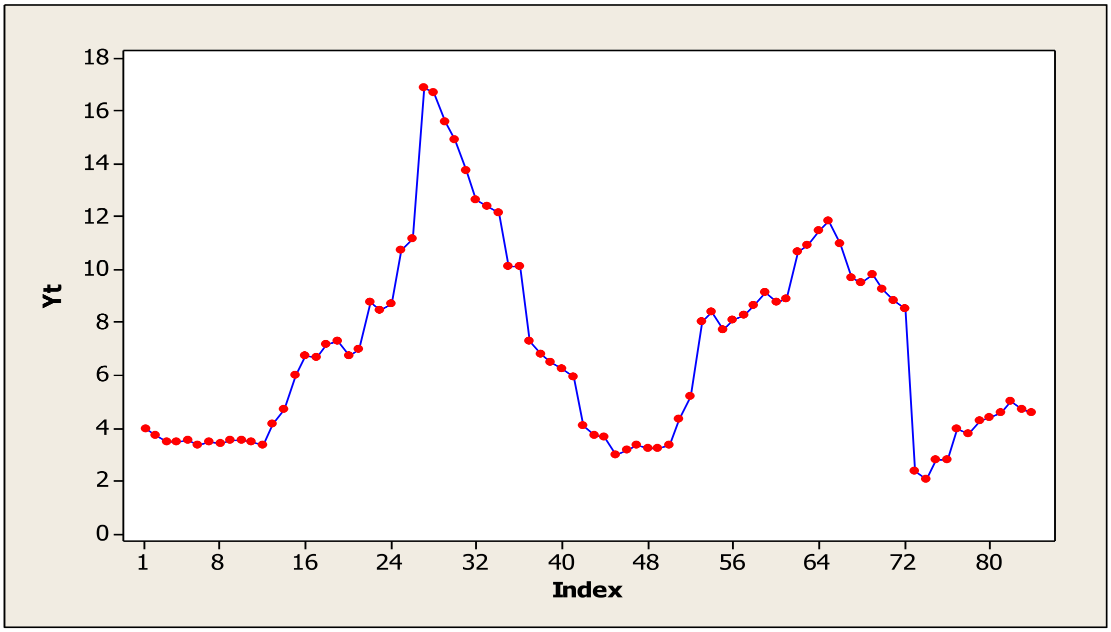

Figure 3: Time series plot of stock market prices Y_t of Julius Berger Nigeria Plc, between the years 2006 to 2012.

The autocorrelation function (ACF) of the series (Y_t) shown in Table 3 and the plot shown in Figure 2 showed a damped cosine wave and decayed slowly from a value of 0.936 at lag one to a value of -0.538 at lag 20, indicating the presence of trend in the series, while the partial autocorrelation function (PACF) in Figure 3 has spike at lag 1 only. The ACF show that the series requires differencing to isolate the trend.

Table 4: ACF and PACF of the original series Y_t, first order differenced series (dY_t) and the residuals (e_t)

k	ACF(Y_t)	PACF(Y_t)	ACF(dY_t)	PACF(d Y_t)	ACF(e_t)	PACF(e_t)
1	0.936	0.936	0.137	0.113	0.137	0.069
2	0.852	-0.185	0.124	0.089	-0.144	0.147
3	0.754	-0.153	-0.096	-0.148	0.022	-0.136
4	0.640	-0.156	0.110	0.128	0.040	-0.015
5	0.536	0.058	0.115	0.105	-0.027	0.046
6	0.420	-0.187	0.038	0.006	-0.042	0.015

7	0.289	-0.194	-0.101	-0.193	0.125	-0.075
8	0.155	-0.126	-0.016	0.032	-0.117	0.117
9	0.029	0.025	-0.150	-0.111	-0.066	-0.087
10	-0.094	-0.135	-0.074	-0.022	-0.086	-0.091
11	-0.202	-0.031	0.018	0.002	0.0197	-0.132
12	-0.302	-0.086	-0.012	0.062	-0.040	0.099
13	-0.387	0.008	-0.096	-0.127	-0.109	-0.030
14	-0.469	-0.184	-0.205	-0.186	-0.044	-0.139
15	-0.536	-0.024	-0.168	-0.080	-0.119	-0.094
16	-0.575	0.048	-0.120	0.018	-0.001	-0.057
17	-0.596	0.004	-0.057	-0.053	-0.065	-0.006
18	-0.602	-0.118	-0.097	-0.122	-0.154	-0.069
19	-0.597	-0.065	-0.247	-0.232	-0.062	-0.171
20	-0.582	-0.008	-0.160	-0.039	0.137	-0.087
21	-0.538	0.146	0.137	0.113	-0.144	0.069

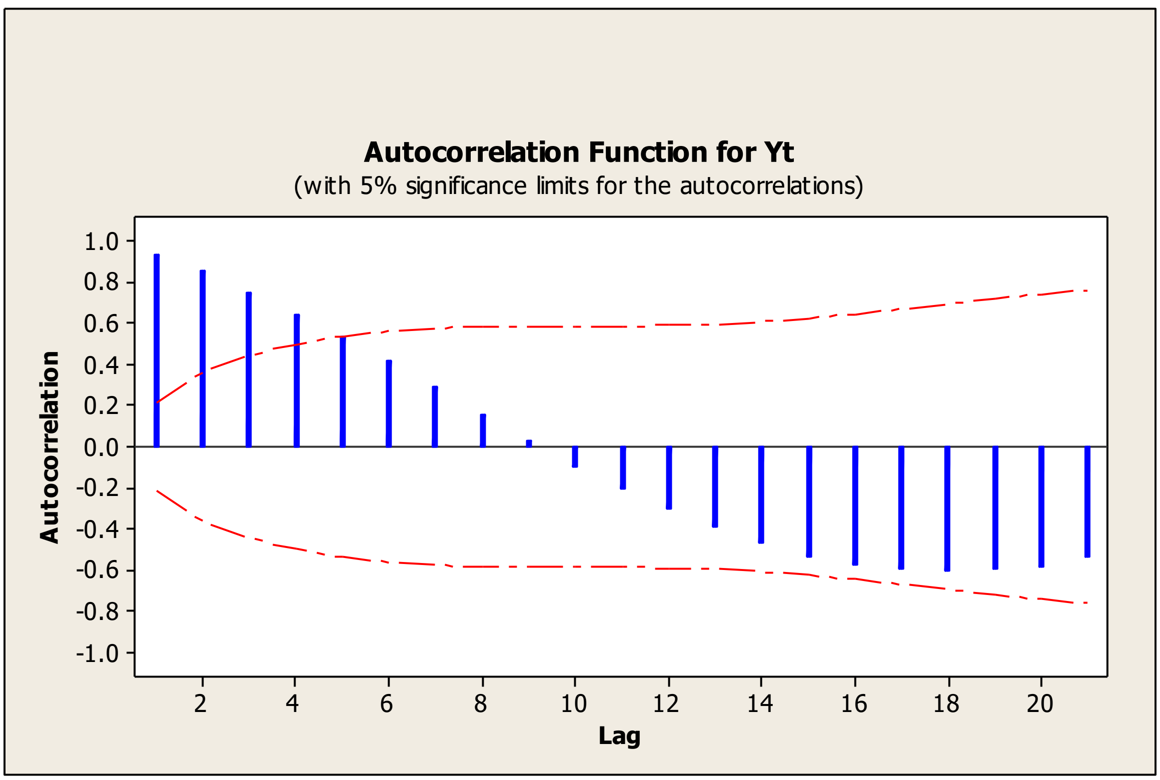

Fig. 4: Autocorrelation function of the series Y_t

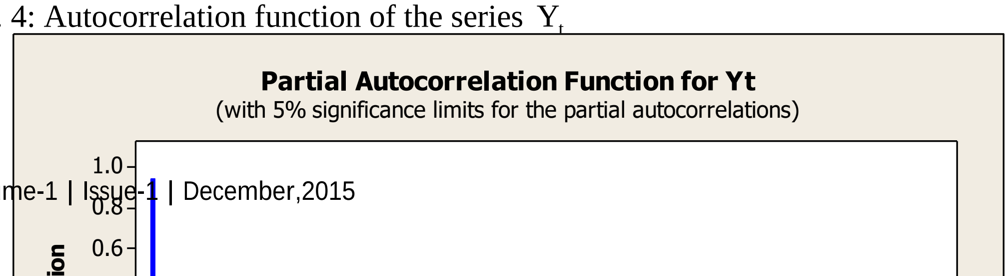

Fig. 5: Partial Autocorrelation function of the series Y_t

The time plot of the differenced series (dY_t) shown in Figure 6 now fluctuate about a line through zero indicating that the trend may have been removed or isolated.

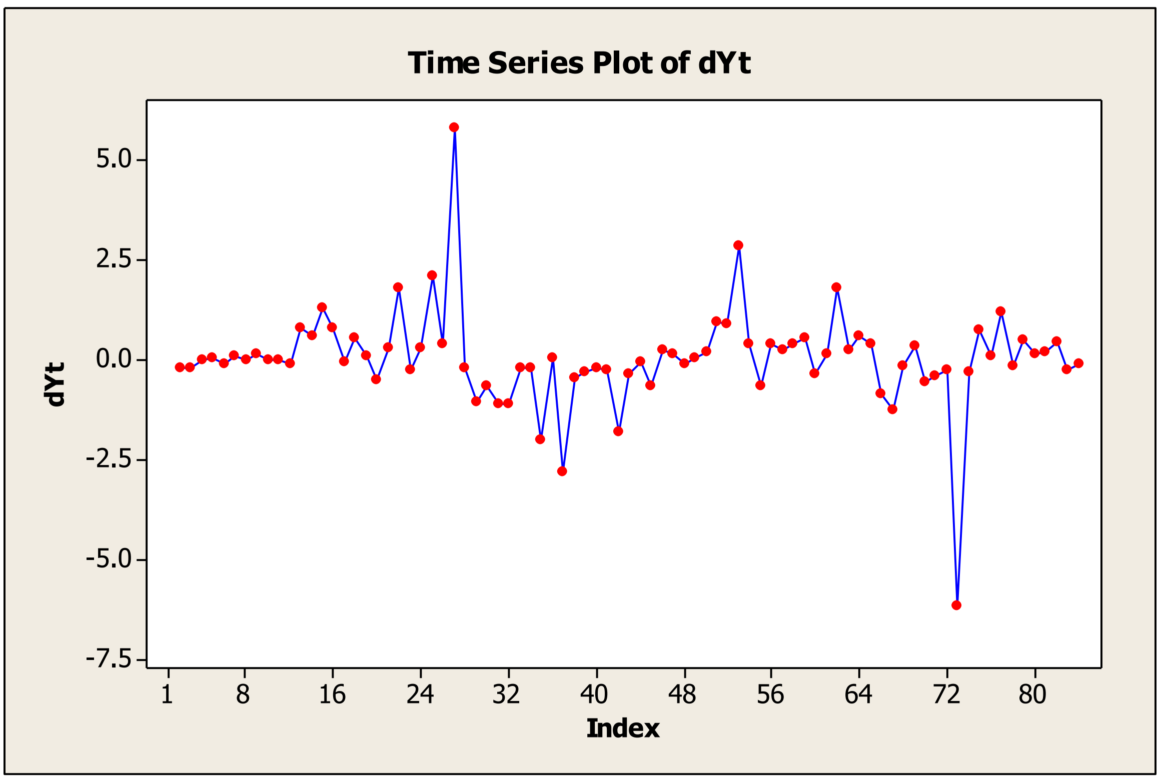

Fig. 6: Time series Plot of the differenced series dY_t

The ACF and PACF of the differenced series (dY_t) shown in Figures 5 and 6 also suggest that the differenced series is stationary with spike at lag 20 of the PACF. This however suggest that the model to be tentatively used is the ARIMA (p,d,q) x $(P,D,Q)_{12}$, with p = d = q = 1, P = Q = 1, D = 0. The model consist of an Autoregressive process of order one [AR (1)], regular differencing of order one or Integrated process of [I(1)], moving average of order one [MA(1)], Seasonal Autoregressive process of order 1 [SAR(1)], and Seasonal Moving Average of order 1 [SMA(1)].

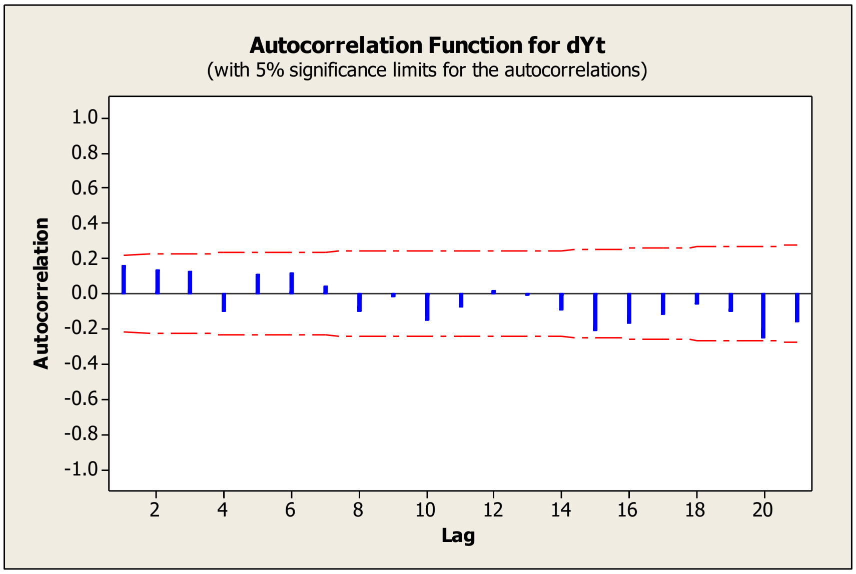

Fig. 7: Autocorrelation Function of the differenced series dY_t

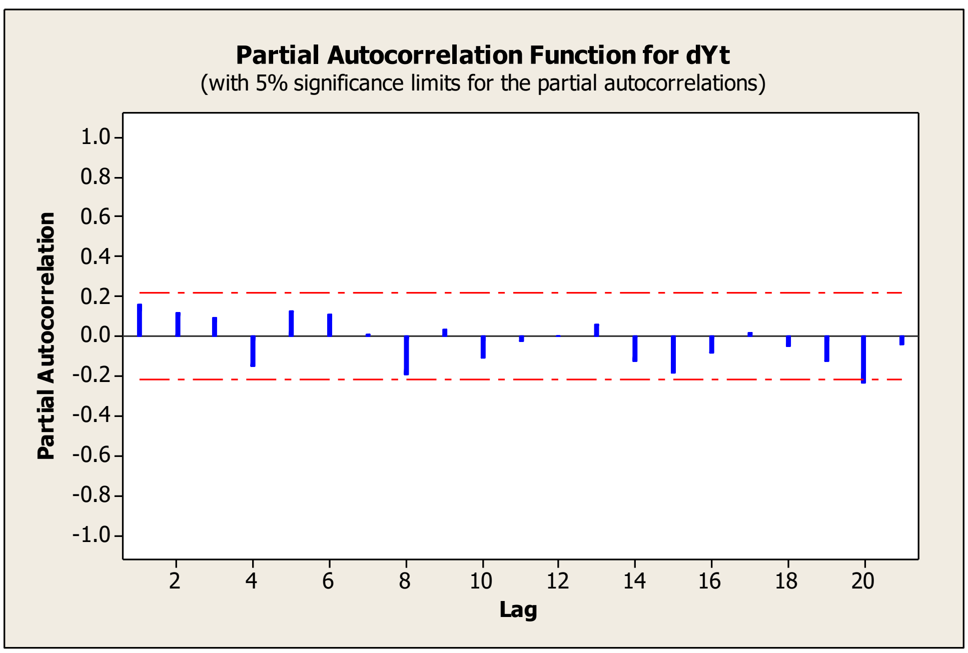

Fig. 8: Partial Autocorrelation Function of the differenced series dY_t

The suggested model ARIMA $(1,1,1) \times (1,0,1)_{12}$, was fitted to the series Y_t and the resultant residuals e_t were evaluated to check the adequacy or otherwise of the fitted model. All the ACF and PACF of the residuals e_t, as shown in Table 4 and Figures 6 and 7, lie with the 95% confidence limits ($\pm\frac{2}{\sqrt{n-1}} = 0.2195$). This indicates that the fitted model is adequate (in terms of residual and ACF and PACF) to describe the pattern in the series.

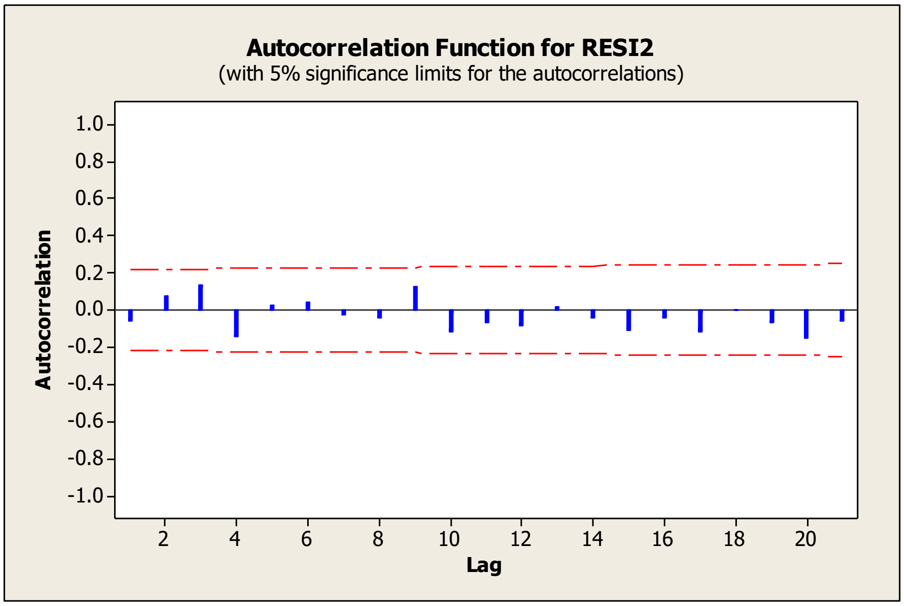

Fig. 9: Autocorrelation function of the Residual e_t

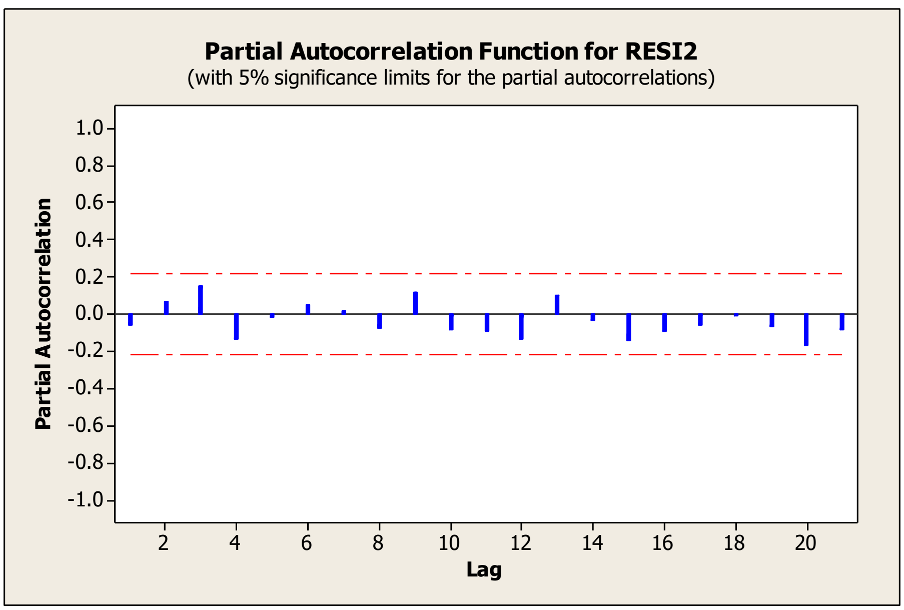

Fig. 10: Partial Autocorrelation function of the Residual e_t

The estimates of the parameter of the selected model, from the MINITAB software package are $\hat{\phi}_1 = 0.7643$, with standard error of 0.2306, $\hat{\Phi} = 0.9709$, with standard error of 0.0554, $\hat{\theta}_1 = 0.5904$, with standard error of 0.2897, $\hat{\Theta} = 0.8110$, with standard error of 0.01147 and constant of 0.00037 with standard error of 0.1332. The p-value associated with the constant indicates that the constant is not significant. Hence, the model is fitted without the constant, and the fitted model is given by;

$$\hat{Y}_t = \hat{\phi}_1 Y_{t-1} + \hat{\Phi} Y_{t-12} + \hat{\theta}_1 e_{t-1} + \hat{\Theta} e_{t-1}$$

Substituting the parameters in the model above gives;

$$\hat{Y}_t = 0.7643Y_{t-1} + 0.9709Y_{t-12} + 0.5904e_{t-1} + 0.8110e_{t-1} \quad (10)$$

FORECASTING

If we denote the forecast made at time for the lead time k by $\hat{Y}_{t_0}(k)$, then the estimate of the forecast function is given by;

$$\hat{Y}_{t_0}(k) = 0.7643Y_{t_0+k-1} + 0.9709Y_{t_0+k-12} + 0.5904e_{t_0+k-1} + 0.8110e_{t_0+k-1} \quad (11)$$

						95% Confidence limit	
lead	$t_0 + k$	Month	Actual Y_{t_0}	Forecast $Y_{t_0}(k)$	$\varepsilon_{t_0}(k)$	Lower	Upper
1	85	January	2.84	3.085	-0.245	0.733	5.436
2	86	February	3.08	3.335	-0.255	-0.291	6.961
3	87	March	3.89	4.220	-0.33	-0.533	8.972
4	88	April	4.31	4.449	-0.139	-1.344	10.241
5	89	May	4.87	4.971	-0.101	-1.793	11.735
6	90	June	4.23	4.648	-0.418	-3.030	12.326
7	91	July	3.26	4.370	-1.110	-4.170	12.911
8	92	August	3.35	4.273	-0.923	-5.083	13.629
9	93	September	3.54	4.329	-0.789	-5.801	14.459
10	94	October	4.42	4.525	-0.105	-6.340	15.391
11	95	November	3.34	4.268	-0.928	-7.300	15.835
12	96	December	3.53	4.151	-0.621	-8.086	16.389

Using the model in Equation (2) with $t_0 = 84$, the MINITAB software package was employed to give the forecast $\hat{Y}_{t_0}(k), k = 1,2,\ldots,12$ for the 12 months of 2013.

S/N	Squared Error	Absolute Error	Absolute Percentage Error
1	0.060	0.245	8.627
2	0.065	0.255	8.279
3	0.109	0.330	8.483
4	0.019	0.139	3.225
5	0.010	0.101	2.074
6	0.175	0.418	9.882
7	1.232	1.110	34.049
8	0.852	0.923	27.552

9	0.623	0.789	22.288
10	0.011	0.105	2.376
11	0.861	0.928	27.784
12	0.386	0.621	17.592
Total	4.403	5.964	172.211

$$\text{Mean Square Error (MSE)} = \frac{4.403}{12} = 0.367$$

$$\text{Mean Absolute Error (MAE)} = \frac{5.964}{12} = 0.497$$

$$\text{Mean Absolute Percentage Error (MAPE)} = \frac{172.211}{12} = 14.351$$

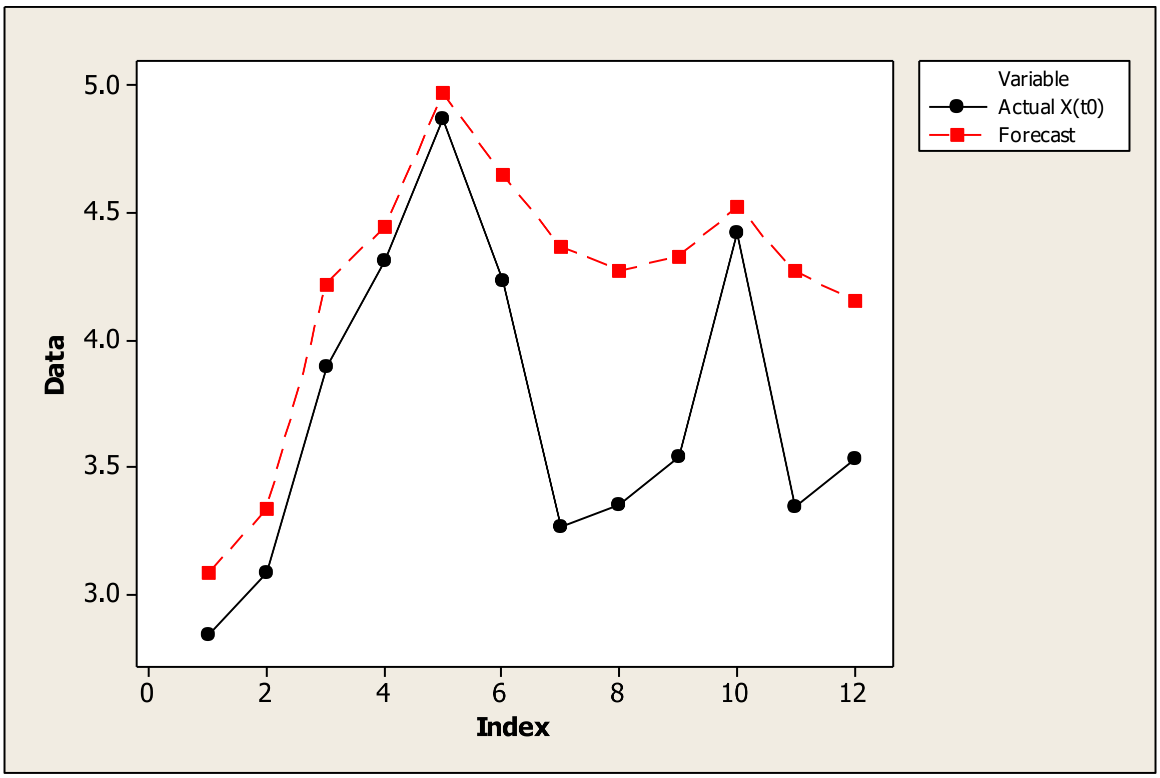

Fig. 11: Plot of Actual values and forecast values of for the year 2013.

The forecast shows that the model did not do well in forecasting the stock market prices of Julius Berger Nigeria limited for the period of 2013.

Conclusion

This research explains fitting an ARIMA Model to Monthly record of stock market prices of Julius Berger Nigeria limited for the period of January 2006 to December 2012 obtained from the website of cash craft asset management limited (a member of Nigerian Stock Exchange), while the 2013 Figures were used to assess the forecasting performance of the fitted model.

Under the model identification, parameter estimation and diagnostic checking, the stock market prices of Julius Berger Nigeria limited was shown to follow the SARIMA (1, 1, 1) x $(1, 0, 1)_{12}$ model.

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